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Refrigeration of an array of cylindrical nanosystems by superfluid helium counterflow

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ABSTRACT

Motivated by the challenge of computer refrigeration, we study the limits set by the transition to quantum turbulence on the cooling of an array of heat-producing cylindrical nanosystems by means of superfluid-helium counterflow. The effective thermal conductivity in laminar counterflow superfluid helium is obtained in channels with rectangular cross section, through arrays of mutually parallel cylinders and in the combined situation of arrays of orthogonal cylinders inside the rectangular channel. The maximum cooling capacity is analyzed on the condition that turbulence is avoided and that the highest temperature does not exceed the lambda temperature.

1. Introduction

Heat transport has always been a topic of interest for applications [1–5]. In superfluid helium (He II) it is also interesting both because of its peculiar ability to flow with very low viscosity and inside very narrow channels, and because of its high thermal conductivity [6–12]. Currently, He II is often used to cool down and keep at very low temperatures the superconducting magnets used in particle accelerators, the measuring instruments assembled in some satellites, and many other cryogenic operations.

Owing to its features, He II will probably find applications in miniaturized devices, too. One of these applications could be the cooling of future quantum computers, i.e., modern devices which aim to use quantum–mechanical phenomena (as for example superposition and entanglement) to perform massively parallel operations on data. In fact, quantum computers require low temperatures to keep, as long as possible, a sufficiently high extent of quantum coherence of the global wavefunction of their constituent qubits.

In the present paper, in order to explore more in depth the theoretical problems arising in superfluid dynamics, we consider the helium flow (and the consequent heat-transport problem related to it) between two parallel planes in the presence of an array of cylinders orthogonal to them (Refer to Fig. 1 for a qualitative sketch of this geometry). From the practical point of view, those cylinders are aimed to model small nanodevices which have to be kept at low temperature by removing the heat per unit time $\dot{Q}_j$ that each of them produces in its operations (as for instance, the computations). In particular, we aim to obtain the practical limits to efficient cooling set by the transition to quantum turbulence. For instance, if one has an array of $M \times (N+1)$ cylindrical devices, each of one dissipating heat at some rate $\dot{Q}$, which will be the maximum total heat we will be able to extract from the system per unit time avoiding the appearance of quantum turbulence? Given $\dot{Q}$, for instance, which will be the maximum value of N we will be able to keep at constant temperature by extracting the dissipated heat? Or given a value of N, which will be the maximum value of $\dot{Q}$ we may manage to extract? And how do these limiting values depend on the radius of the cylinders, the

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Nomenclature

Latin letters

a	channel's width
A	channel's transversal section
b	channel's height
c	semi distance between the axes of two consecutive cylinders
C	nondimensional parameter
C_v	specific heat per unit mass at constant volume
d	channel's smallest size
K	friction coefficient
K_{eff}	effective thermal conductivity
l	channel's longitudinal length
L	vortex length density of quantum turbulence
M	total number of columns of cylinders
n	number of columns from a given axis
N	total number of rows of cylinders
p	pressure
$\mathbf{q}$	local heat-flux vector
q	modulus of local heat-flux vector
$\dot{Q}$	heat per unit time
R	radius of cylinder
S	entropy per unit volume
T	temperature
$\mathbf{v}$	barycentric fluid speed
$\bar{\mathbf{v}}_n$	speed of normal component
$\bar{\mathbf{v}}_s$	speed of superfluid component
V_{ns}	the relative velocity $\bar{\mathbf{v}}_n - \bar{\mathbf{v}}_s$

Greek letters

α	Reynolds-number function
β	coefficient of the rate of destruction of vortices per unit volume and time
η	shear viscosity
Θ	nondimensional temperature
κ	quantum of circulation
λ_1	thermal conductivity
ρ	total mass density
ρ_n	mass density of normal component
ρ_s	mass density of superfluid component
τ_1	relaxation time of local heat flux
ϕ	nondimensional ratio
φ	aspect ratio of cross section
ω_2	second-sound speed
ω'	Reynolds-number function

Subscripts

crit	critical
cyl	cylinder
eff	effective
fin	final
in	initial
lam	laminar
q	quantum
tot	total
turb	turbulent
visc	viscous
λ	lambda point

Superscripts

(bath)	bath
c	cylinder
s	superfluid
(tot)	total

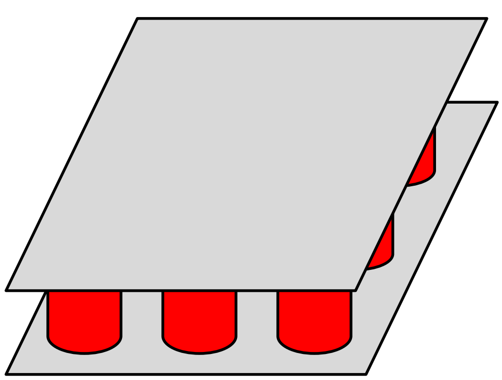

Fig. 1. Sketch of an array of transversal cylinders symmetrically distributed over a channel filled by superfluid helium. The channel has a rectangular transversal section with high aspect ratio. The cylinders are located orthogonally to the plates and to the heat flow.

separation of the cylinders, and the separation between the parallel plates?

The paper runs as following. In Section 2 we write the basic equations allowing to describe heat transfer in superfluid helium. In Section 3 we derive in the linear regime the effective thermal conductivity of He II in a channel with a rectangular cross section and with an array of transversal cylinders inside it. In Section 4 we assume that all the internal cylinders are heat sources, and study a possible way to refrigerate the system. In Section 5 the main results of the present paper are summarized, with emphasis on the effects of turbulence. In an Appendix, we provide the mathematical details of the derivation of the effective thermal conductivity mentioned above.

2. Basic equations for heat transfer in superfluid helium

In the steady-state one-fluid model with $\mathbf{v}$ as the barycentric fluid speed, and with the local heat flux $\mathbf{q}$ as internal variable, for zero net-mass flow, the heat transfer in He II can be described by the model equations

$$\nabla \cdot \mathbf{v} = 0 \tag{II.1a}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{II.1b}$$

$$\nabla p - \eta\left(\nabla^2\mathbf{v} + \frac{\nabla^2\mathbf{q}}{ST}\right) = \mathbf{0} \tag{II.1c}$$

$$\lambda_1\nabla T - \frac{\eta\lambda_1}{S}\left(\nabla^2\mathbf{v} + \frac{\nabla^2\mathbf{q}}{ST}\right) = -\mathbf{q}(1 + \tau_1 KL) \tag{II.1d}$$

$$L^{3/2}\left[\beta\kappa L^{1/2} - \left(\frac{q\alpha}{ST} - \frac{\kappa\omega'\beta}{d}\right)\right] = 0 \tag{II.1e}$$

once nonlinear terms have been neglected [13–16].

In Eqs. (II.1) S means the entropy per unit volume, p pressure, T temperature, q is the modulus of the local heat flux, τ_1 is the relaxation time of $\mathbf{q}$, and λ_1 and η can be interpreted as the thermal conductivity and the shear viscosity, respectively, when applied to a classical fluid [13]. The two material functions λ_1 and τ_1 can be related to the second-sound speed w_2 by means of the relation