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Coupled heat conduction and multiphase change problem accounting for thermal contact resistance

Daniel Weisz-Patrault

LMS, École Polytechnique, CNRS, Université Paris-Saclay, 91128 Palaiseau, France

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ABSTRACT

In this paper, heat conduction coupled with multiphase changes are considered in a cylindrical multilayer composite accounting for thermal contact resistance depending on contact pressures and roughness parameters. A numerical simulation is proposed using both analytical developments and numerical computations. The presented modeling strategy relies on an algorithm that alternates between heat conduction accounting for volumetric heat sources and a multiphase change model based on non-isothermal Avrami's equation using the isokinetic assumption. Applications to coiling process (winding of a steel strip on itself) are considered. Indeed, phase changes determine the microstructure of the final material and are responsible for residual stresses that create flatness defects. A Finite Element modeling is used for validating the presented solution and numerical results are presented and discussed.

1. Introduction

1.1. Heat conduction problem in a multilayer composite

A cylindrical multilayer composite is considered as shown in Fig. 1a. Contact pressures, that can vanish if contact is not ensured, are known as well as heterogeneous initial temperature and phase fields. This paper aims at developing a modeling strategy enabling effective simulation of the unsteady heat conduction problem, accounting for multiphase changes that occur during cooling. Since applications to axisymmetric coil cooling problems are considered, a radial model is derived, although the circumferential direction could have been addressed. Heat fluxes along the axial direction are neglected despite the fact that the temperature field slightly evolves along the axial direction because of contact pressure distribution and boundary conditions at cylinder edges. Therefore, a one-dimensional problem is obtained and applied several times at different axial locations. Industrial temperature measurements, recorded with an infrared camera during coil cooling, are presented in Fig. 1b. The axisymmetric assumption is well verified.

The heat conduction modeling strategy relies on analytical developments. This choice leads to reasonable computation times that enable parametric studies. Analytical solutions for multilayer composites have already been studied. For instance, the analytical solution of the unsteady heat conduction problem without heat sources in a 1D multilayer composite has been established in plates, cylinders and spheres by De Monte [1,2]. An extension has been proposed by De Monte [3] for a 2D bi-layer composite in Cartesian coordinates. More recently Singh et al. [4], Jain et al. [5] extended this solution for a cylindrical 2D multilayer composite (radial and circumferential directions) considering time-independent heat sources. The same problem has been solved for spherical configurations by Jain et al. [6]. The radial 1D heat conduction problem for a bi-layer composite has been recently solved by Li and Lai [7] using Laplace transforms. All these works do not consider heat sources or deal with time-independent heat sources. In this paper, a coupling with multiphase change problems is considered, thus time and space dependent heat sources should be taken into account.

Necati Özışık [8] obtained a very elegant solution for the 1D heat conduction problem in multilayer composites (plate, cylinder and sphere) considering time and space dependent heat sources. A compact closed-form solution, very suitable for analytical heat sources, is proposed. However, in this paper heat sources are given numerically as outputs of the multiphase change problem, with a given time and space discretization. It should be noted that a direct numerical computation of the closed-form solution proposed in [8] would not be efficient in terms of computation times. Indeed, the latter solution relies for each time t on a primitive denoted by $\int_0^t g_n^*(t')\mathrm{d}t'$, where functions $g_n^*(t')$ are defined for each time t' as a sum of several numerical integrals over the strip thickness. The time discretization can be very fine, so that phase changes are correctly described. Since t' lies between 0 and t the time discretization for the integrand $g_n^*(t')$ should be even finer and

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E-mail address: weisz@lms.polytechnique.fr

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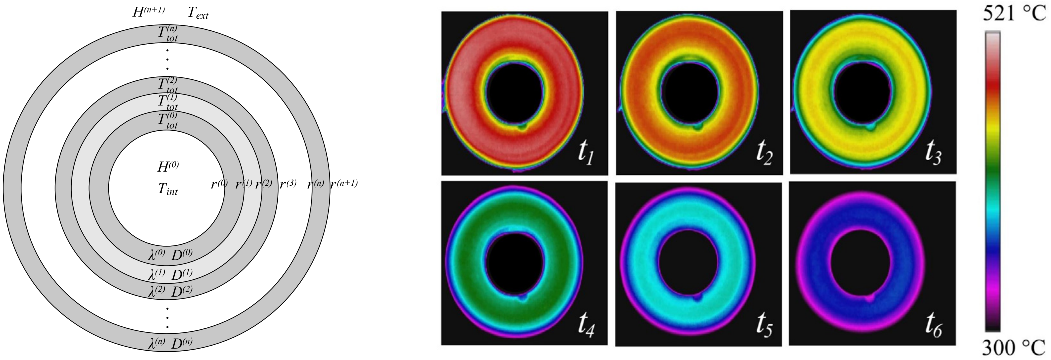


(a) Multilayer composite (b) Axisymmetric coil cooling problem

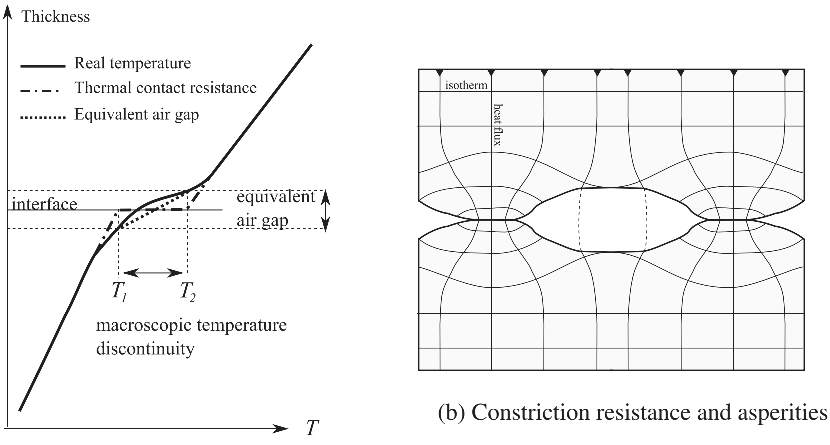
Fig. 1. Heat conduction modeling.

numerical evaluations of the proposed solution are long for the considered applications. The solution presented in [8] relies on an expansion of heat sources into a series of the form (25). In this paper, an alternative idea is to expand heat sources into time-dependent exponential series by using data fit procedures. The obtained solution avoids long numerical computations of $g_n^*(t')$ and mainly rely on matrix inversion. One could have used the closed-form solution in [8] at this point, however calculations are done under the assumption that heat fluxes vanish at the inner radius. The solution could be fairly easily adapted for a surrounding temperature at the inner radius identical to those at the outer radius, but more significant changes should be made if the surrounding temperature at the inner radius is different from those at the outer radius. Therefore, the analytical solution derived in the following enables to consider different surrounding temperatures at inner and outer surfaces. Mathematical developments rely on quite similar ideas developed in [8] on the one hand and data fit procedures on the other hand.

1.2. Thermal contact resistance

Furthermore, contact pressures between layers are assumed to be known. Therefore, thermal contact resistance can be evaluated depending on roughness profiles. All previously cited analytical solutions deal with continuous temperature and heat flux through each interface, which corresponds to perfect thermal contact. Indeed, orthogonality relations that are used to fulfill the initial condition do not hold if thermal contact resistances are introduced. In this paper, this difficulty is overcome by assuming perfect contact conditions, however thin insulating air gaps between steel layers are introduced, in order to model an equivalent thermal contact resistance. The appropriate air gap thickness is determined as a function of thermal contact resistance by ensuring that the heat flux flowing through the imperfect interface is the same as the average heat flux flowing through the equivalent air gap. The macroscopic temperature discontinuity at the interface that defines thermal contact resistance is presented in principle in Fig. 2a. The modeling strategy relying on temperature and flux continuities, thin insulating air gaps are introduced in order to smooth interface discontinuities without pretending a better description of the real contact. Indeed, the latter smoothing is rather artificial and relies completely on established thermal contact resistance that characterizes stochastic properties of real contact topology.

Determination of thermal contact resistance between rough surfaces has been intensively studied. Kaza [9] presents an interesting review of the field and based on contact theories and experiments a thermal contact resistance that depends on roughness parameters and contact pressure is proposed. This latter thermal contact resistance is used in this paper. The real contact area is only a small proportion of the nominal contact area according to roughness of both surfaces. There remain asperities that are filled usually with a gas or a liquid if contact is lubricated. Early papers of Shlykov and Ganin [10] and Cooper et al. [11] describe a calculation method of thermal contact resistance for steel, depending on contact pressure and roughness. Two thermal contact resistances considered as parallel resistances are identified, namely constriction resistance and resistance of asperities. On the one hand, constriction resistance describes the fact that the heat flux narrows



(b) Constriction resistance and asperities

(a) Temperature at the interface

Fig. 2. Thermal contact resistance.