International Journal of Heat and Mass Transfer 104 (2017) 627–633

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Fast collapse of a vapor bubble

P.A. Pavlov

*Institute of Thermophysics, Ural Branch of the Russian Academy of Sciences, Amundsen Street 107a, 620016 Ekaterinburg, Russia*

ARTICLE INFO

*Article history:*
Received 28 June 2016
Received in revised form 26 August 2016
Accepted 29 August 2016

*Keywords:*
Evaporation
Density jump
Adiabatic process
Condensation
Stream bubble
Adiabatic compression
Heat-mass transfer

ABSTRACT

The dynamics of collapse of a vapor bubble under the action of an instantaneously applied external pressure has been calculated. In the initial state the bubble is in thermal and mechanical equilibrium with the surrounding liquid. The bubble compression causes vapor condensation and establishment of the pressure of saturated vapor in the bubble at the interface temperature. An adiabatic increase in the temperature is presumably observed in the body of the bubble. The temperature field and the heat fluxes close to the interface have been calculated. The mass flux from the bubble has been found from the heat balance. The pressure field in the liquid was calculated for a nonviscous and incompressible liquid. As a result, a system of equations has been obtained for calculating the vapor pressure in the bubble, the bubble radius and the compression rate. The role of the condensation process in the bubble dynamics has been elucidated. The pressure and the temperature in the final stage of collapse of a bubble have been found.



## 1. Introduction

There are a large number of mathematical models of the evolution of a single spherical bubble with different degrees of complexity and specification level. In models based on the Rayleigh-Plesset equation it is possible to obtain practically useful analytical solutions for the problem on the bubble collapse [1,2]. The results obtained are well-known and published in educational literature [3,4]. It is generally recognized that the model of adiabatic compression [5] is valid in the case of fast collapse of gas-filled bubbles. In this case the pressure field near the bubble is inhomogeneous, and the phenomenon of implosion may be observed. By another well-known model [3], the bubble is believed to be empty and isolated, so the pressure in the bubble $p''$ is equal to zero. These two models give the complete spectrum of extreme notions of the bubble dynamics without allowance for evaporation or condensation processes. It is generally assumed that the details of the process of gas or vapor compression are not essential for the dynamics of bubble collapse [4]. Nevertheless, there is still a rather problematic and interesting challenge, that of a rigid account of vapor condensation in the bubble. The phenomenon of rapid condensation is much more difficult for calculation than the problem of fast evaporation. The reason for this difficulty lies in the fact that it is impossible to present a condensation front by analogy with an evaporation front at a high pressure. As is well known [6], intense evaporation is simulated by a pressure jump on a Knudsen layer, and, as a result, it is easy to get the evaporation rate for vapor at any pressure. The front of rapid condensation might be formally simulated by a jump of a pressure drop on a Knudsen layer. But since shock rarefaction waves are unstable, such an approach is erroneous. Rigorous kinetic theories of evaporation are suitable only for low-pressure vapor. In practice, waves of a rapid pressure drop in the vapor phase are unlikely to be realized, and perhaps for this reason calculations of condensation at the molecular level are still in the stage of discussions [7].

The role of the condensation process is especially significant in the case of bubble collapse in high-temperature heat-transfer agents, and also in extremely superheated (metastable) liquids. This is connected with the fact that at a high absolute temperature the temperature increment caused by the adiabatic compression is comparatively large. As a result, one can observe a large excess of the vapor temperature over that of the interface. Numerical evaluations of the condensation rate give ground to assume that in most cases the limiting factor of the condensation process is the heat transfer [2]. Analytical solutions here are too complicated, and computer solutions do not give a visual demonstration of the general laws of the process. Below is given an economic combined model of the collapse of a vapor bubble which contains an analytical calculation and a simple numerical solution. Well-known methods of calculation of boundary-value problems [8] have been used to find heat fluxes at an interface with allowance for convective terms. Here it proved to be necessary to take into account the specific character of solution of boundary-value problems with a phase transition at an interface. Since at saturation pressure and

*E-mail address:* p-pavlov@mail.ru

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.08.097
0017-9310/

**Nomenclature**

*Latin letters*

| | |
|---|---|
| $a$ | thermal diffusivity, $m^2/s$ |
| $b$ | vapor flow rate, m/s |
| $c$ | specific heat capacity, J/(kg K) |
| $j$ | vapor flow density, kg/($m^2$ s) |
| $t$ | evaporation time, time of bubble compression, s |
| $m$ | vapor mass, kg |
| $T$ | temperature, K |
| $T_S$ | interface temperature, K |
| $T''$ | vapor temperature, K |
| $r$ | radius from the center of the bubble, m |
| $R$ | bubble radius, m |
| $p$ | pressure, MPa |
| $p_S$ | saturated vapor pressure at the interface temperature, MPa |
| $y \equiv p''/p_0$ | relative vapor pressure in the bubble |
| $L$ | heat of evaporation, J/kg |
| $u = U\sqrt{\rho_0''/p_0''}$ | dimensionless rate |
| $U$ | rate of change of the vapor radius, m/s |
| $x$ | distance from the interface in the liquid, m |
| $W = \rho'' c_p'' d\,T''/dt$ | power of heat release under adiabatic compression, $W/m^3$ |

*Greek letters*

| | |
|---|---|
| $\theta(r,t) = T(r,t) - T_0$ | excess temperature, K |
| $\Gamma(\varepsilon)$ | Euler gamma function |
| $\rho''$ | vapor density, $kg/m^3$ |
| $\rho_S''$ | saturated vapor density at the interface temperature, $kg/m^3$ |
| $y$ | adiabatic index |
| $\lambda$ | thermal conductivity, W/(m·K) |
| $\sigma$, $\varepsilon$, $\phi$ | adjustment parameters |

*Subscripts*

| | |
|---|---|
| $S$ | of the liquid surface, belonging to the saturation line |
| $\infty$ | temperature deep in liquid, pressure far from the bubble |
| Index 0 | initial value |

*Primes*

no primes liquid properties
double primes vapor properties

a rigidly prescribed interface temperature the vapor on the wall condenses instantly, calculation of the temperature field in a bubble proved to be analytically possible. The pressure field in the liquid is determined by the Euler equation [3]. Such a forced simplification is used because the role of liquid viscosity and compressibility in problems on the dynamics of bubbles is often insignificant and has been described in close detail [9].

## 2. Simulation of the condensation process

The suggested model of combined calculation of bubble dynamics is based on the bubble mass balance equation and the Euler equation. The mass balance equation is solved with allowance for the heat balance in the bubble – liquid system. The collapse rate is determined by the liquid hydrodynamics and the equal-in-volume vapor pressure in the bubble. When a macrobubble is compressed by an external pressure, the vapor temperature additionally increases considerably at the cost of the local growth of pressure ("implosion" phenomenon).

Below is considered a complicated process which includes the collapse determined by the rate of removal of the vapor condensation heat and the hydrodynamics that determines the pressure in the bubble. The condensation coefficient was subsequently taken to be equal to unity.

Let us take the initial pressure in a bubble–liquid system $p_0''$ to be equal to the pressure of saturated vapor at an interface temperature $T_s$, the initial vapor density $\rho_0''$ is also taken on the saturation line. The bubble is assumed to be macroscopic, therefore we do not take into account the interfacial tension. At the initial instant the external liquid pressure at infinity is set, and a pressure field begins to form in the vicinity of the bubble. As the bubble is collapsing, the interface temperature $T_s$ changes owing to heat exchange and vapor condensation. We suppose that in the bubble core (far from the interface) the process of vapor compression is adiabatic, therefore the vapor density varies by the power law

$$\rho''/\rho_0'' = (p''/p_0'')^{1/\gamma} \tag{1}$$

The exponent is subsequently assumed arbitrary. If $\gamma = 1$, then we have an isothermal process at a varying pressure (vapor in a thermostat). If $\gamma = 0$, the pressure becomes constant. Actually in an adiabatic process $\gamma = 1 + 2/i$ ($i$ is the number of the molecule degrees of freedom).

Let us introduce the density of the mass flux into the bubble caused by condensation or evaporation at the interface $j = \dot{m}/(4\pi R^2)$, where $\dot{m}$ is the rate of change of the vapor mass in the bubble. The vapor mass in a bubble of radius $R$ is equal to $m = 4\pi R^3 \rho_0'' y^{1/\gamma}/3$, therefore from the balance of the bubble mass we can write an equation for the relative vapor pressure $y = p''/p_0''$ at the present instant of time $t$:

$$\dot{y} = \frac{3\gamma j}{R\rho_0''} y^{1-1/\gamma} - y\frac{3\gamma \dot{R}}{R} \tag{2}$$

This equation is valid only for collapse without condensation in the body of vapor. At adiabatic compression of an initially equilibrium bubble the temperature in the vapor body of most heat-transfer agents increases more rapidly than the condensation temperature, therefore volume condensation is excluded. One can make sure of that by comparing an increment in the condensation temperature with an increase in the pressure by the well-known approximation

$$T_S(p) - T_S(p_0) \approx (d\ln p_S/dT)^{-1} \ln(y) \tag{3}$$

with an increment in the gas temperature $\theta = T'' - T_0$ at adiabatic compression

$$\theta = \left[y^{(\gamma-1)/\gamma} - 1\right] T_0. \tag{4}$$

At an arbitrary exponent $\gamma$ the validity of Eq. (2) must be verified (Fig. 1).

With low-temperature heat-transfer agents a situation may arise where the saturation temperature proves to be higher than that in the body of vapor. Therefore the process of compression may be accompanied by volume condensation. Next we assume that the vapor temperature in the central part of the bubble $T''$ is higher than the interface temperature $T_S$. To solve Eq. (2), the initial bubble radius $R_0$ and the initial rate of its change $\dot{R}_0$ are