International Journal of Heat and Mass Transfer 104 (2017) 644–649

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Stability of a confined swirling annular liquid layer with heat and mass transfer

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ARTICLE INFO

Article history:
Received 21 June 2016
Received in revised form 31 August 2016
Accepted 31 August 2016

Keywords:
Linear stability analysis
Confined annular layer
Swirling liquid layer
Interface heat and mass transfer

ABSTRACT

A linear temporal stability of confined swirling annular liquid layers involving heat and mass transfer at the gas–liquid interface is presented in this paper. A normal-mode stability analysis that includes the effect of both swirling and heat transfer is performed. The flow in a gas-center coaxial swirl injector gives rise to the flow configuration explored in this work. The effects of various non-dimensional parameters on the instability of the flow are discussed. The heat transfer at the interface has been characterized by introducing a heat flux ratio between the conduction heat flux and the evaporation heat flux. The heat and mass transfer at the interface are found to destabilize the flow. Increasing confinement destabilizes the flow, which is opposite to the condition without heat and mass transfer.

1. Introduction

Annular liquid layers or sheets are encountered in various industry applications, e.g. the disintegration of liquid stream and film cooling. For an annular liquid sheet, the interplay of pressure, aerodynamic, centrifugal, and surface tension forces gives rise to surface oscillations. Under certain conditions these oscillations get amplified to cause instability. The stability of a moving annular liquid layer or sheet is of importance to the performance and reliability of the equipment involving the annular liquid layer flows. There are numerous literatures on the instability [13,7,4,15] and breakup morphology [14,3,16] of annular liquid sheets. However, most of these studies did not take the effect of heat and mass transfer into account. There are many situations when the effect of mass and heat transfer across the interface play an essential role in determining the instability characteristics of the flow.

Heat and mass transfer were first taken into account in the stability analysis of the flow system by Hsieh [8,9]. It is found that heat and mass transfer can decrease the growth rate for Rayleigh–Taylor instability. The effect of heat and mass transfer were also considered in the Kelvin–Helmholtz instability. Nayak and Chakraborty [11] found that heat and mass transfer has a destabilizing effect on the Kelvin–Helmholtz instability of a cylindrical interface. Adham-Khodaparast et al. [1] found that heat and mass transfer play a deleterious effect on Kelvin–Helmholtz instability for a planar flow. Recently, Mohanta et al. [10] studied the stability of coaxial jets confined in a tube with heat and mass transfer. They found increasing heat and mass transfer at the interface stabilizes the flow to small as well as very large wave numbers.

To our knowledge, no stability analyses of confined swirling annular liquid layers involving heat and mass transfer have been performed. The objective of the present study is to investigate the effect of heat and mass transfer at the interface on the stability of confined swirling annular layers. Effects of various non-dimensional parameters on the stability are explored in this study. A non-dimensional heat flux ratio of conduction-to-evaporation heat transfer, characterizing the effect of heat and mass transfer at the interface, has been identified in this work. The present results show that heat and mass transfer plays a significant role in destabilizing the flow.

2. Problem formulation

The configuration shown in Fig. 1 is a swirling annular liquid layer confined in a tube, with gas phase in the center. The thickness of the liquid layer is h. The radius of gas–liquid interface is R. In this study, the analysis is developed in a cylindrical coordinate (z, r, θ). The coordinate system is chosen so that the z axis is parallel to the direction of the flow motion. The model consists of a cylindrical gas jet with velocity (u_g, v_g, w_g), surrounded by co-flowing annular liquid layer with velocity (u_l, v_l, w_l), where u, v, and w represent the velocity in z, r, and θ directions respectively. Only the effect of rigid rotation around z axis is considered, hence the angular

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.08.113
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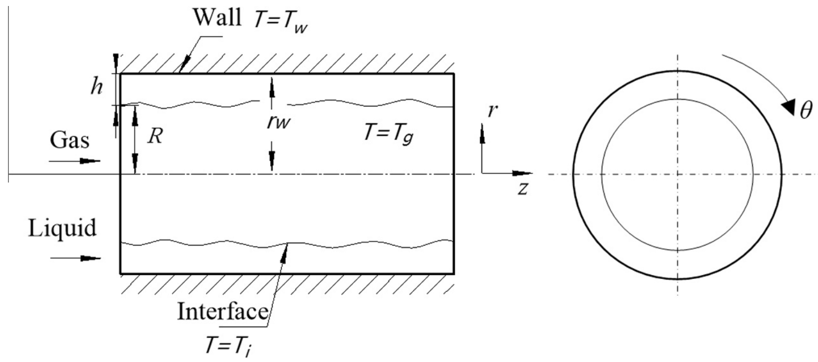

Fig. 1. Schematic of the model.

velocity of rotation for liquid layer is defined as β. The liquid and gas phase have the constant density ρ_l and ρ_g. The surface tension of liquid was σ, and the effect of viscosity is neglected in the present work. The bulk temperature of the center gas phase is T_g, the interface temperature is considered to be T_i and the wall is at a temperature of T_w. For the case with phase change T_i would be the saturation temperature.

When disturbances set in, the interface deforms and deviates away from the equilibrium state. The flow field is disturbed with the perturbed flow velocity and pressure superimposing on the base flow velocity and pressure

$$[u_i, v_i, w_i, p_i] = [\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{p}_i] + [u'_i, v'_i, w'_i, p'_i] = [\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{p}_i] + [\hat{u}_i, \hat{v}_i, \hat{w}_i, \hat{p}_i]\exp(ikz + \omega t) \quad (1)$$

where the over-bar denotes the steady components and the apostrophe stands for the perturbation components. The perturbation components are assumed to be wave-type in perturbation normal modes, defined mathematically by a real wave number k, and a complex frequency $\omega = \omega_r + i\omega_i$.

The linearized governing equations for liquid phase can be obtained by substituting Eq. (1) into the continuity and momentum equations as follows

$$\hat{v}_l + r\frac{\partial \hat{v}_l}{\partial r} + ikr\hat{u}_l = 0 \quad (2)$$

$$(\omega + ik\bar{u}_l)\hat{v}_l - 2\beta\hat{w}_l = -\frac{1}{\rho_l}\frac{\partial \hat{p}_l}{\partial r} \quad (3)$$

$$(\omega + ik\bar{u}_l)\hat{w}_l + 2\beta\hat{v}_l = 0 \quad (4)$$

$$(\omega + ik\bar{u}_l)\hat{u}_l = -\frac{1}{\rho_l}ik\hat{p}_l \quad (5)$$

$\hat{w}_l$ can be expressed as Eq. (6) according to Eq. (4)

$$\hat{w}_l = \frac{-2\beta}{\omega + ik\bar{u}_l}\hat{v}_l \quad (6)$$

Also, it can be obtained through Eq. (5) that

$$\frac{\omega + ik\bar{u}_l}{ik}\frac{\partial \hat{u}_l}{\partial r} = -\frac{1}{\rho_l}\frac{\partial \hat{p}_l}{\partial r} \quad (7)$$

Substituting Eqs. (6) and (7) into Eq. (3), we obtain

$$\left[1 + \frac{4\beta^2}{(\omega + ik\bar{u}_l)^2}\right]\hat{v}_l = \frac{1}{ik}\frac{\partial \hat{u}_l}{\partial r} \quad (8)$$

Substituting Eq. (8) into Eq. (2) gives

$$\frac{\partial^2 \hat{u}_l}{\partial(\lambda r)^2} + \frac{\partial \hat{u}_l}{\partial(\lambda r)}\frac{1}{\lambda r} - \hat{u}_l = 0 \quad (9)$$

where $\lambda = k\sqrt{f},\ f = 1 + \frac{4\beta^2}{(\omega + ik\bar{u}_l)^2}$.

Then the solution of Eq. (9) can be written as

$$\hat{u}_l = AI_0(\lambda r) + BK_0(\lambda r) \quad (10)$$

The governing equations given in Eq. (3) are subjected to the following boundary conditions at the wall

$$v'_l = 0,\ \text{and}\ \eta = 0,\ \text{at}\ r = r_w = R + h \quad (11)$$

where $\eta = \hat{\eta}\exp(ikz + \omega t)$ denotes the displacements of the interface.

The linearized mass balance at the interface is given by

$$\rho_l\left(v'_l - \frac{\partial \eta}{\partial t} - \bar{u}_l\frac{\partial \eta}{\partial z}\right) = \rho_g\left(v'_g - \frac{\partial \eta}{\partial t} - \bar{u}_g\frac{\partial \eta}{\partial z}\right) \quad (12)$$

The linearized energy balance at the interface is expressed as Eq. (13), and the detailed derivation of Eq. (13) can be found in Mohanta et al. [10]

$$v'_l - \frac{\partial \eta}{\partial t} - \bar{u}_l\frac{\partial \eta}{\partial z} = \Lambda\eta S'(0) \quad (13)$$

where $\Lambda = \frac{\Delta T k_l}{\rho_l \bar{u}_l L R}$ represents the ratio between conduction heat flux from the wall to the interface and the evaporation heat flux at the interface, $\Delta T = T_i - T_w$. The derivation of $S'(0)$ can be found in Appendix.

Substituting Eqs. (8) and (10) into Eqs. (11) and (13) gives

$$B = A\frac{I_1(\lambda R_w)}{K_1(\lambda R_w)} \quad (14)$$

$$\frac{1}{i\sqrt{f}}[AI_1(\lambda R) - BK_1(\lambda R)] = [\omega + ik\bar{u}_l + \Lambda S'(0)]\hat{\eta} \quad (15)$$

Arranging $l_1 = \omega + ik\bar{u}_l + \Lambda S'(0)$, and combining Eqs. (11) and (15), it is obtained that

$$A = \frac{i\sqrt{f}l_1\hat{\eta}K_1(\lambda R_w)}{K_1(\lambda R_w)I_1(\lambda R) - I_1(\lambda R_w)K_1(\lambda R)} \quad (16)$$

$$B = \frac{i\sqrt{f}l_1\hat{\eta}I_1(\lambda R_w)}{K_1(\lambda R_w)I_1(\lambda R) - I_1(\lambda R_w)K_1(\lambda R)} \quad (17)$$

Then $\hat{p}_l$ will be expressed as follow by combining Eqs. (5) and (10)

$$\hat{p}_l = \frac{-\rho_l}{ik}(\omega + ik\bar{u}_l)[AI_0(\lambda R) + BK_0(\lambda R)] \quad (18)$$

For gas phase, the linearized governing equations can be written as:

$$\hat{v}_g + r\frac{\partial \hat{v}_g}{\partial r} + ikr\hat{u}_g = 0 \quad (19)$$