Marangoni convection in a thin layer of nanofluid: Application to combinations of water or ethanol with nanoparticles of alumina or multi-walled carbon nanotubules

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A B S T R A C T

A linear stability analysis for the onset of Marangoni convection in a horizontal layer of a nanofluid heated from below is investigated. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The lower boundary of the layer is assumed to be a rigid surface at fixed temperature while the top boundary is assumed to be a non-deformable free surface cooled by convection to an exterior region at a fixed temperature. The boundaries of the layer are assumed to be impenetrable to nanoparticles with their distribution being determined from a conservation condition. The linear analysis uses spectral methods based on the expansion of eigenfunctions as Chebyshev series. Stability boundaries for temperature and volume fraction Marangoni numbers are obtained for nanofluids on either distilled water or ethanol with either alumina or multi-walled carbon nanotubules as nanoparticles.

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1. Introduction

Recently, a new class of fluid called a "nanofluid" opened a new dimension in the study of thermal instability. A nanofluid refers to a base fluid that contains particles of maximum size 100 nm. Common heat transfer fluids used as base fluids include water and various organic fluids such as kerosene, ethylene glycol and ethanol, while the nanoparticles used include metallic or metallic oxide particles such as copper, copper oxide, alumina (Al_2O_3) and also multi-walled carbon nanotubules (MWCNTs).

The heat transfer properties of fluids can be increased substantially by the presence of solid particles suspended in the fluid. Many researchers [1–6] have found an increase in the thermal conductivity of ordinary fluids using nanoparticles.

Buongiorno [5] conducted an extensive study of nanofluids and derived the conservation equations of a non-homogeneous equilibrium model of a nanofluid. This model incorporated the effects of Brownian diffusion and thermophoresis. On the basis of this model, studies were conducted by many authors (e.g. Tzou [7,8]; Nield and Kuznetsov [9,10]; Kuznetsov and Nield [11,12]). A book by Das et al. [13] and a review article by Das and Choi [14] covered a wide area on heat transfer in nanofluids.

The flow of nanofluid is of great interest in numerous areas of modern science, engineering and technology, the chemical and nuclear industries and bio-mechanics. Nanofluids are not naturally occurring but are synthesized in laboratories. The choice of base fluid and particle combination depends on the application for which the nanofluid is intended. Such fluids are used in industry as coolants, lubricants, for the delivery of drugs, in heat exchangers and in micro-channel heat sinks among other applications [15–17].

The convective instability of nanofluids has been studied by several authors. Kim et al. [18] investigated analytically the convective instability driven by buoyancy and heat transfer characteristics of nanofluids. Tzou [7,8] studied the thermal instability of nanofluids (Rayleigh–Benard convection) and showed that nanofluids are less stable than regular fluids. Recently Nield and Kuzentsov [9,10] and Kuznetsov and Nield [11,12] studied the onset of convection in a horizontal layer of a porous medium saturated by a nanofluid using a model for the nanofluid that incorporates the effects of Brownian motion and thermophoresis. Yadav et al. [19,20] considered the linear stability of Rayleigh–Benard convection in nanofluids with and without rotation, and obtained a sufficient condition for the existence of overstability. The onset of convection in a porous medium saturated with a viscoelastic nanofluid was studied by Sheu [21] who showed that oscillatory instability is possible for both top and bottom heavy nanoparticle

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distributions. Nield and Kuzentsov [22] studied the onset of double-diffusive convection in a nanofluid layer. The effect of internal heating on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid was studied by Yadav et al. [23]. They found that the internal heat source has a destabilizing effect on the system. Ramesh and Rana [24] considered the effect of rotation on a horizontal layer of porous medium saturated by a nanofluid. The effect of magnetic field on the thermal instability of nanofluids has been discussed by Gupta et al. [25], Yadav et al. [26] and Mahajan and Arora [27]. These authors showed that the presence of a magnetic field has a stabilizing influence. Rotating nanofluids have been discussed by Yadav et al. [28], Agarwal and Bhadauria [29] and others.

The general trend for base fluids is that their surface tension decreases with increasing temperature. For nanofluids, however, surface tension depends on both temperature and nanoparticle concentration. Experimental studies for different nanofluids have reported that the presence of nanoparticles causes surface tension to be increased [30–32], to be unchanged [33,34] or to be decreased [4,35].

Marangoni convection for the base fluid is induced by the dependence of surface tension on temperature. The earliest work on Marangoni instability in a fluid layer heated from below was performed by Pearson [36] who showed that, rather than being a buoyancy driven flow, Benard cells are primarily induced by the surface tension gradients resulting from temperature variations across the free surface. Nield [37] showed that for layers of depth at most 1 mm the buoyancy effect can safely be neglected for most liquids. The findings of Pearson and Nield have been extended and polished by many workers for deformable and non-deformable surfaces (e.g. Takashima [38,39]; Benguria and Depassier [40]; Wilson [41]; Shivakumara et al. [42]; Hashim and Arifin [43]; Shivakumara et al. [44]). For non-deformable surfaces, Marangoni instability ensues through the mechanism of stationary convection. However for deformable surfaces overstability is possible under some conditions.

The earliest work on Rayleigh-Marangoni instability in a layer of fluid heated from below was performed by Nield [37] who showed that buoyancy and surface tension reinforce each other and are tightly coupled. The work of Nield [37] has been extended and polished by many researchers for deformable and non-deformable surfaces (e.g. Sarma [45]; Garcia-Ybarra et al. [46]; Benguria and Depassier [40]; Perez-Garcia and Carneiro [47]; Hennenberg et al. [48]; Hashim and Arfin [43]; Zhao et al. [49]). However, a survey of the literature revealed that no research has been conducting with regard to the stability of Marangoni convection in nanofluids. Therefore the object of the present study is to investigate this problem for several types of nanofluid.

2. Mathematical formulation

Consider an infinite horizontal layer of an incompressible nanofluid confined between the planes $x_3^* = 0$ and $x_3^* = d$ of a rectangular Cartesian system of coordinates with position vector $\mathbf{x}^* = x_1^* \mathbf{e}_1 + x_2^* \mathbf{e}_2 + x_3^* \mathbf{e}_3$ with $\mathbf{e}_3$ directed vertically upwards. Let $T^*(t^*, \mathbf{x}^*)$ and $\phi^*(t^*, \mathbf{x}^*)$ denote respectively the Kelvin temperature and volume fraction of nanoparticles in the fluid at time t^* and position $\mathbf{x}^*$. On $x_3^* = 0$ the nanofluid is assumed to rest on a rigid boundary which is maintained at a constant Kelvin temperature T_0, whereas on $x_3^* = d$ the motion of the nanofluid is driven by the thermocapillary effect of the surface tension of the nanofluid, say $\gamma(T^*, \phi^*)$, but otherwise the surface is non-deformable and loses heat by convection to an environment at constant temperature T_1.

2.1. Model equations

This analysis will assume that the layer of nanofluid is very thin ($d \leqslant 1$ mm), and consequently buoyancy effects can be ignored in the statement of the momentum equations. Following the work of Buongiorno [5], Tzou [7,8] and Nield and Kuznetsov [9,10], the governing equations for a nanofluid in the absence of chemical reactions and gravity, but in the presence of thermophoresis are

$$\frac{\partial V_j^*}{\partial x_j^*} = 0, \tag{1}$$

$$\rho(\bar{\phi})\left(\frac{\partial V_j^*}{\partial t^*} + V_k^* \frac{\partial V_j^*}{\partial x_k^*}\right) = -\frac{\partial P^*}{\partial x_j^*} + \mu(\bar{\phi})\frac{\partial^2 V_j^*}{\partial x_k^* \partial x_k^*}, \tag{2}$$

$$\frac{\partial \phi^*}{\partial t^*} + V_k^* \frac{\partial \phi^*}{\partial x_k^*} = -\frac{1}{\rho_s}\frac{\partial J_k^*}{\partial x_k^*}, \tag{3}$$

$$(\rho C)(\bar{\phi})\left(\frac{\partial T^*}{\partial t^*} + V_k^* \frac{\partial T^*}{\partial x_k^*}\right) = -\frac{\partial q_k^*}{\partial x_k^*} + h_s(T^*)\frac{\partial J_k^*}{\partial x_k^*}, \tag{4}$$

where $\mathbf{V}^* = V_j^* \mathbf{e}_j$ denotes the nanofluid velocity, P^* denotes hydrostatic pressure and $\bar{\phi}$ denotes the average volume fraction of nanoparticles. The Boussinesq approximation has been used in the formulation of Eqs. (2)–(4), namely that variations in temperature and volume fraction of nanoparticles within the layer are sufficiently small that the material properties of the nanofluid may be treated as constant in all equations with the exception of buoyancy terms when these are present. Typically the values assigned to these constant coefficients are computed by averaging the respective material property across the layer. In the case of nanofluid, the density, dynamic viscosity and specific heat at constant pressure are given by the respective constitutive equations

$$\rho(\phi) = \phi \rho_s + (1 - \phi)\rho_f, \tag{5}$$

$$\mu(\phi) = \mu_f(1 - \phi)^{-5/2}, \tag{6}$$

$$(\rho C)(\phi) = \phi \rho_s C_s + (1 - \phi)\rho_f C_f \tag{7}$$

in which ρ_f and C_f denote respectively the density and specific heat of the base fluid, ρ_s and C_s denote respectively the density and specific heat of the nanoparticle material, and μ_f is the dynamic viscosity of the base fluid. Expression (6) is Brinkman's model [50] for dynamic viscosity. Thus the appropriate specifications of the constant material properties in Eqs. (2)–(4) are

$$\frac{1}{d}\int_0^d \rho(\phi)\,dx_3^* = \frac{1}{d}\int_0^d \left(\phi \rho_s + (1-\phi)\rho_f\right)dx_3^* = \rho(\bar{\phi}) \tag{8}$$

$$\frac{1}{d}\int_0^d \mu(\phi)\,dx_3^* = \frac{1}{d}\int_0^d \mu_f(1-\phi)^{-5/2}\,dx_3^* \approx \mu(\bar{\phi}) \tag{9}$$

$$\frac{1}{d}\int_0^d (\rho C)(\phi)\,dx_3^* = \frac{1}{d}\int_0^d \left((\rho C)_s \phi + (\rho C)_f(1-\phi)\right)dx_3^* = (\rho C)(\bar{\phi}). \tag{10}$$

While expressions (8) and (10) are exact results, expression (9) is a mean value result for integrals.

The vector $\mathbf{J}^* = J_k^* \mathbf{e}_k$ in Eqs. (3) and (4) denotes the diffusive mass flux of nanoparticles and is the sum of contributions from Brownian and thermophoresis diffusions, namely

$$\mathbf{J}^* = -\rho_s \left(D_B \frac{\partial \phi^*}{\partial x_j^*} + \frac{D_T}{T_0}\frac{\partial T^*}{\partial x_j^*}\right)\mathbf{e}_j, \tag{11}$$

where T_0 is the Kelvin temperature of the lower boundary of the nanofluid layer, and D_B and D_T (assumed constant) represent respectively the Brownian and thermophoretic diffusion coefficients with respective constitutive specifications

$$D_B = \frac{kT_0}{3\pi\mu_f d_s}, \quad D_T = \frac{\mu_f}{\rho_f}\left(\frac{0.26\kappa_f}{2\kappa_f + \kappa_s}\right)\bar{\phi}. \tag{12}$$