International Journal of Heat and Mass Transfer 104 (2017) 703–717

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A spectral element model for nonhomogeneous heat flow in shallow geothermal systems

Noori BniLam [*], Rafid Al-Khoury

Faculty of Civil Engineering and Geosciences, Computational Mechanics, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands

ARTICLE INFO

Article history:
Received 21 June 2016
Received in revised form 16 August 2016
Accepted 17 August 2016

Keywords:
Borehole heat exchanger
GHP
GSHP
Spectral element method
FFT
Nonhomogeneous heat flow

ABSTRACT

A comprehensive spectral element formulation for nonhomogeneous heat flow in a shallow geothermal system consisting of a borehole heat exchanger embedded in a multilayer soil mass is introduced. The spectral element method is utilized to solve the governing heat equations in the borehole heat exchanger and the soil mass simultaneously using the fast Fourier transform, the eigenfunction expansion, the Fourier Bessel series and the complex Fourier series, together with the finite element method. Only one spectral element is necessary to describe heat flow in a homogeneous domain. For a nonhomogeneous multilayer system, the number of spectral elements is equal to the number of layers. The proposed spectral element model combines the exactness of the analytical methods with an important extent of generality in describing the geometry and boundary conditions of the numerical methods. Verification examples illustrating the model accuracy, and numerical examples illustrating its capability to simulate multilayer systems are given. Despite the apparent rigor of the proposed model, it is robust, computationally efficient and easy to implement in computer codes.

1. Introduction

Heat flow in nonhomogeneous domains consisting of components with different physical properties is central among numerous engineering applications. Heat flow in pipes, heat exchangers, solids and layered domains are only few examples of such applications. Solution of the involved heat equations vary between analytical, semi-analytical and numerical, depending on the complexity of the problem. In this publication, we present a semi-analytical methodology for solving transient conductive-convective heat flow in nonhomogeneous domains, which might consist of multiple components with different geometrical and physical properties. The proposed methodology is applicable to a wide range of engineering applications, but the focus here is on shallow geothermal systems.

A shallow geothermal system, known as geothermal heat pump (GHP) or ground source heat pump (GSHP), is a source of renewable energy that utilizes the earth heat energy from shallow depths for heating and cooling of buildings. It works by circulating a fluid (refrigerant), mostly water with antifreeze solution, through a closed loop of polyethylene U-tube pipe that is inserted in a borehole in a soil mass. The borehole is filled with grout to fix the polyethylene pipe and to ensure a good thermal interaction with the soil.

The borehole heat exchanger is a slender heat pipe with dimensions of the order of 30 mm in diameter for the U-tube, and 150 mm in diameter and 100 m in length for the borehole. The circulating fluid in the U-tube collects heat from the surrounding soil mass via convection-conduction heat flow mechanisms. Physically, the heat flow mechanism in such a system is well understood, but computationally, and in spite of the bulk of existing models, still creeping due to the combination of the slenderness of the boreholes heat exchangers and the involved thermal convection. This combination of geometry and physics constitutes the main source of computational challenges in this field. Consequently, several geometrical and physical simplifications have been introduced in order to circumvent this problem and obtain feasible solutions. All known solution techniques, such as analytical, semi-analytical and numerical, have been utilized for this purpose. Nevertheless, in spite of the versatility of the numerical methods, analytical and semi-analytical solutions are yet preferable because of their comparatively little demands on computational power and ease of use in engineering practice.

Most of the current analytical and semi-analytical models for heat flow in geothermal heat pumps are based on the work of Carslaw and Jaeger [1] for modeling heat flow in finite, semi-infinite and infinite domains subjected to point, line, plane and cylindrical

---

[*] Corresponding author.
E-mail addresses: n.h.n.bnilam@tudelft.nl, noori.alnoori@gmail.com (N. BniLam).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.08.055
0017-9310/

heat sources. In these models, the BHE detailed composition and heat transfer mechanisms are totally ignored and considered as a constant heat source. Gu and O'Neal [2] introduced an analytical model simulating transient heat flow in a composite domain subjected to a constant heat source, resembling U-tubes surrounded by grout, and a soil mass bounded by a far field boundary. They utilized the eigenfunction expansion to solve the governing partial differential equation. Based on Gu and O'Neal's approach, Lamarche and Beauchamp [3] solved the composite domain problem using Laplace transform. They solved both forward and inverse Laplace transforms analytically. Bandyopadhyay et al. [4] solved the same problem using dimensionless equations, and employed the Gaver–Stehfest numerical algorithm for solving the inverse Laplace transform.

Eskilson and Claesson [5] diverged from the Carslaw and Jaeger solutions and introduced a semi-analytical model for ground source heat pumps that approximates heat flow in the borehole heat exchangers by two interacting channels conveying a circulating fluid in the vertical axis and embedded in an axisymmetric soil mass. Heat flow in the channels is assumed steady state convective, and in the soil, transient conductive. They utilized Laplace transform to solve the heat equations of the channels, and the explicit forward difference method to solve the heat equations of the soil mass. Zeng et al. [6] solved the same problem but using dimensionless heat equations for the channels.

Marcotte and Pasquier [7] introduced a semi-analytical model for a transient pseudo convection using the fast Fourier transform for discretizing the time domain, and the cubic spline for interpolating results obtained at selected spatial samples. They utilized the principle of superposition to simulate the response to multiple heat fluxes. Javed and Claesson [8] solved Gu and O'Neal's problem using a similar pseudo convective approach.

Recently, notable attempts have been introduced to account for the inevitable presence of multiple soil layers in shallow geothermal systems. Raymond and Lamarche [9] analyzed the effect of multiple layers in determining the thermal parameters from the thermal response test (TRT) results. They adopted an analytical computer code for transient well flow in layered aquifer systems to describe conductive heat transfer in shallow geothermal systems constituting multiple layers and subjected to a variable heat injection rate. The Laplace transform is utilized to solve the system of partial differential equations describing heat flow in the layered system. Abdelaziz et al. [10] extended the finite line heat source solution to a multiple segment finite line heat source resembling a layered soil profile. The temperature of the heterogeneous domain is obtained by summing up the temperature of the typical homogeneous domain with that obtained due to the presence of other layers. The latter is calculated by assuming a composite system constituting smeared thermal parameters, described as a function of the relative distances of the layers from the point of interest.

Despite the appeal of these endeavors, current analytical and semi-analytical models are in general limited in describing the geometry and physics of heat flow in shallow geothermal systems. The main shortcomings are twofold: (1) Not all the details of heat transfer mechanisms in the BHE are taken into consideration. The BHE is considered as a line or cylindrical heat source, ignoring the heat flow in its components and their thermal interactions. (2) The soil mass is in general considered infinite or semi-infinite. Even if a multilayer system is adopted, the BHE is assumed a line or a cylindrical heat source with a constant or a variable heat flux. Here, these two shortcomings are treated.

In a previous work, Al-Khoury [11,12] introduced a semi-analytical model for transient conductive-convective heat flow in shallow geothermal systems based on the spectral analysis. The model is valid for a semi-infinite domain, where the system can extend to infinity in the vertical and the radial directions. No soil layers with different physical parameters are permitted. However, it is likely that the soil mass surrounding the BHE consists of several layers with different thermal interaction effects. To tackle this, here, the spectral element method is utilized to formulate a semi-analytical model for shallow geothermal systems consisting of a single U-tube borehole heat exchanger embedded in a layered soil mass.

The spectral element method (SEM) is a semi-numerical (semi-analytical) technique which combines the spectral analysis method, basically the discrete Fourier transform, with the finite element method. In the literature, the spectral element method corresponds to two different techniques. The first corresponds to the work introduced by Patera [13], and the second corresponds to the work introduced by Doyle [14]. Patera's spectral element method deals mainly with spectral formulations in the spatial domain. In this, the domain is discretized into a number of elements, and the field variable in each element is represented by a high-order Lagrangian interpolation through Chebyshev collocation points. It is thus a finite element method with high degree piecewise polynomial basis functions capable of producing high order accuracy.

Doyle's spectral element method, on the other hand, deals mainly with a spectral formulation in the temporal domain. It is a combination of the spectral analysis method, the dynamic stiffness method and the finite element method. In this work, we adopt the temporal SEM of Doyle. For more account of the historical and theoretical background of the spectral element method, see Lee [15].

The spectral element method is an elegant technique used mainly for solving wave propagation problems. One of the important features of this method is that its formulation leads to a set of equations, similar to that of the conventional finite element method. The fundamental difference, however, is that the spectral element stiffness matrix is exact and frequency dependent. Due to the exact formulation of the system, one element is sufficient to describe a whole homogenous domain. For a nonhomogeneous domain consisting of several layers or members, the number of the spectral elements is equal to the number of the involved layers or members. This feature significantly reduces the size of the problem, and rendering this method computationally very efficient.

The spectral element method discretizes a space–time field variable into a frequency domain and an eigenmode domain. The discretization of the time domain to the frequency domain is done using the fast Fourier transform (FFT) algorithm, and the discretization of the spatial domain to the eigenmode domain is done using the eigenfunction expansion. The general solution of the system can be obtained by summing over all significant frequencies and eigenvalues.

In this paper, we formulate a two-node spectral element for transient conduction–convection heat flow in a single U-tube borehole heat exchanger embedded in a layered soil mass. A detailed modeling approach is given hereafter.

2. Modeling approach

A shallow geothermal system, particularly a geothermal heat pump, consists basically of two thermally interacting domains: the borehole heat exchanger and the soil mass.

Upon operating the geothermal heat pump, the temperature in the soil mass arises as a result of the thermal interaction with the borehole heat exchanger. The temperature in borehole heat exchanger, on the other hand, arises from an inlet fluid temperature coming from a heat pump, and the thermal interaction with the soil mass.