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Inclined magnetic field and heat source/sink aspects in flow of nanofluid with nonlinear thermal radiation

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ABSTRACT

This article addresses the melting heat transfer is stagnation point flow of nanofluid towards a stretching surface with nonlinear thermal radiation. Heat transfer phenomenon also takes into account heat generation/absorption. Newly constructed mass flux condition is employed. Nanofluid model consists of Brownian motion and thermophoresis. Fluid is electrically conducting in the presence of applied an inclined magnetic field. The governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by using appropriate transformation. The convergent series solution are worked out by employing homotopic procedure. Behaviors of different physical parameters on the velocity, temperature, concentration, skin friction coefficient and local Nusselt number are analyzed.

1. Introduction

The fluid flow analysis involving the exothermic/endothermic chemical reaction is based upon internal heat changes. The heat generation or absorption deeply effects temperature distribution within the fluid and consequently the rate of deposition of the particle changes. Some examples of such phenomenon are for nuclear reactors, electronic chips and semiconductor wafers. Also during the past few decades the concept of nanofluids is found quite useful in order to overcome the issue of low thermal conductivity of some traditional liquids such as oil, ethylene glycol, water and many others. Obviously the nanofluid is due to mixing of nano meter sized particles into a traditional liquid. Several theoretical and experimental attempts and review papers have been presented for flow analysis of nanofluids under various aspects. Very recently Abbas et al. [1] examined heat generation effect in the hydromagnetic flow of nanofluid induced by a curved stretching sheet. Bodewadt flow of nanofluids caused by stretching disk is explored by Mustafa et al. [2]. Flow of dusty nanofluid due to an exponentially stretching surface is studied by Sandeep et al. [3]. Hayat et al. [4] analyzed flow of MHD nanofluid bounded by a nonlinear stretching surface. Ziaei-Rad et al. [5] discussed MHD flow of nanofluid over a permeable surface. Khan et al. [6] numerically computed three dimensional flow of nanofluid by a convectively heated surface. Forced convection flow of nanofluid over a stretching surface is explored by Sheikholeslami et al. [7]. Das [8] examined partial slip effect in flow of nanofluid bounded by a non-permeable surface. Mixed convection flow of nanofluid over a sheet with slip condition is studied by Hsiao [9]. Thermal radiation effect in stretched flow of ferro-fluid is presented by Zeeshan et al. [10]. The heat transfer in MHD stagnation point flow of nanofluid over a stretching/shrinking surface is investigated by Nandy and Mahapatra [11]. The authors in this attempt developed the analysis in presence of velocity slip, convective conditions and heat generation/absorption. Parsa et al. [12] studied the boundary layer flow over a stretching sheet with magnetic field and internal heat generation/absorption. Alsaedi et al. [13] examined the stagnation point flow of nanofluid towards a permeable stretched surface with convective boundary conditions and internal heat generation/absorption. Pavithra and Gireesha [14] explored the boundary layer flow of Dusty fluid over a stretching surface with internal heat generation/absorption. Mukhopadhyay and Vajravelu [15] performed the analysis for the influence of transpiration in the unsteady MHD two-dimensional flow of Maxwell fluid past a stretching surface with heat generation/absorption. The study of heat generation/absorption effects in the forced convection flow over a plate placed in porous medium is reported by Mandal and Mukhopadhyay [16]. Shehzad et al. [17] formulated the three-dimensional flow of an Oldroyd B fluid in the presence of variable thermal conductivity and heat generation/absorption. Sheikh and Abass [18] reported the effects of thermophoresis and heat

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generation/absorption in magnetohydrodynamic flow over an oscillatory stretching surface with chemically reactive species. Inclined magnetic field and heat source/sink effects in flow of third grade fluid towards an exponentially stretching surface is analyzed by Hayat et al. [19].

Melting heat transfer analysis seems quite relevant in the area of magmasolidification and the melting of permafrost and silicon wafer process. Bachok et al. [20] examined the impact of melting heat transfer in the stagnation point flow towards a stretching/shrinking surface. Hayat et al. [21] considered the melting phenomenon for the double diffusive convective flow of Maxwell fluid near a stagnation point. The magnetohydrodynamic stagnation point flow of Jeffrey fluid towards a semi-infinite stretching sheet with melting effects, viscous dissipation and Joule heating is discussed by Das and Zheng [22]. Hayat et al. [23] explored the melting heat transfer in stagnation point flow of Powell-Eyring fluid over a linearly stretched surface. Hayat et al. [24] further considered the melting effects in the boundary layer stagnation point flow of couple stress fluid past a stretching surface. Gorla et al. [25] employed Runge-Kutta and Shooting methods to inspect the melting heat transfer in boundary layer flow of nanofluid. Hayat et al. [26] dealt with melting heat transfer in the stagnation point flow of Jeffrey fluid past a stretching sheet with Soret and Dufour effects. Characteristics of magnetic field and melting heat transfer in stagnation point flow of Tangent-hyperbolic fluid is reported by Hayat et al. [27]. Yacob et al. [28] analyzed the characteristics of melting phenomenon in boundary layer stagnation point flow of micropolar liquid over a stretching/shrinking surface. Gireesha et al. [29] examined the behavior of melting heat transfer in MHD boundary layer stagnation-point flow of nanofluid over a stretching surface. Hayat et al. [30] explored the MHD flow of Cu-water nanofluid with viscous dissipation and Joule heating in the presence of melting heat transfer.

The aforementioned studies shows that flows with nonlinear thermal radiation are not widely investigated. Very little is reported for stretched flow of viscous fluid. Even such attempt for non-Newtonian fluid further narrowed down. No doubt thermal radiation is useful in the design of equipment, gas turbines and several propulsion devices for aircraft missiles, nuclear plants, space vehicles and satellites. Fluid flow and radiative nonlinear heat transfer towards stretching sheet was discussed by Cortell [31]. Shehzad et al. [32] studied the three-dimensional flow of Jeffrey nanofluid in the presence of nonlinear thermal radiation. Sakiadis flow with nonlinear Rosseland thermal radiation has been examined by Pantokratoras and Fang [33]. Unequal diffusivities instance of homogeneous heterogeneous reactions within viscoelastic fluid flow in the presence of induced magnetic-field and nonlinear thermal radiation is analyzed by Animasaun et al. [34]. MHD three-dimensional flow of nanofluid with velocity slip and nonlinear thermal radiation is analyzed by Hayat et al. [35].

Up till now no attempt has been made for the analysis of melting heat transfer in stagnation point flow of nanofluid towards a stretching surface with nonlinear thermal radiation. Our main objective here is to venture in this regime. Heat generation/absorption effect is also considered. New condition of mass flux for nanofluid is imposed. Homotopic approach [36–48] is implemented for the construction of convergent solutions of the momentum, energy and concentration equations. The velocity, temperature and nanofluid concentration profiles are analyzed graphically for various parameters of interest. Performances of Skin friction coefficient and Nusselt number are also computed graphically. A comparative study is also shown with the previous published results. The solutions obtained by HAM are preferred than the numerical solutions in view of the following points. (i) HAM gives the solutions within the domain of interest at each point while the numerical solutions hold only for a set of discrete points in the domain. (ii) Algebraically produced approximate solutions require less effort and having a reasonable amount of accuracy when compared to numerical solution which is always handy for the scientist, an engineer or an applied mathematician. (iii) Although most of the scientific packages required some initial guesses for the solution are not generally convergent. In such conditions approximate solutions can offer better initial guess that can be readily advanced to the exact numerical solution in a limited iterations. Finally an approximate solution, if it is analytical, is most pleasing than the numerical solutions.

2. Mathematical model

We analyzed the steady two-dimensional stagnation point flow of an incompressible nanofluid past towards a permeable stretching sheet. It is presumed that the external flow velocity is $u_e(x) = ax$ and the stretching sheet velocity is $u_w(x) = cx$, where a and c are the positive constants. Strength of inclined magnetic field is B_0. The effect of melting heat transfer is considered. Here T_∞ and C_∞ are the ambient temperature and concentration fields with $T_\infty > T_m$. We also consider nonlinear thermal radiation and heat generation/absorption effects. Contribution of thermophoresis and Brownian motion of nanoparticles is taken into account. The governing boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e(x)\frac{\partial u_e(x)}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho}(u_e(x)-u)\sin^2\psi, \tag{2}$$

$$\begin{aligned}u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}={}&\frac{k_f}{(\rho c_p)_f}\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)+\frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\\&-\frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y}+\frac{Q_0}{(\rho c_p)_f}(T-T_m),\end{aligned} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The relevant boundary conditions for the problems are

$$\begin{aligned}&u(x,y)=u_w(x)=cx,\ \ v(x,y)=0,\ \ T=T_m,\\&D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0\ at\ y=0,\\&u(x,r)\to u_e(x)=ax,\ \ T\to T_\infty,\ \ C\to C_\infty\ as\ y\to\infty,\end{aligned} \tag{5}$$

where u and v denote the components of velocity in the x and y directions respectively, σ the electrical conductivity, $\nu = (\mu/\rho)_f$ the kinematic viscosity, μ_f the dynamic viscosity, ψ the angle of inclination, T and T_m the fluid and melting surface temperatures respectively, ρ_f the fluid density, $(c_p)_f$ the fluid specific heat, ρ_p the particles density, $(c_p)_p$ the particles specific heat, k_f the thermal conductivity, $\tau = (\rho c_p)_p/(\rho c_p)_f$ the ratio between the effective nanoparticle material heat capacity and fluid heat capacity, Q_0 the coefficient of heat generation and absorption, q_r the radiative heat flux, C the concentration of the fluid, D_B and D_T are the Brownian motion and thermophoretic coefficients.

The condition of melting heat transfer related to the problem is given by [30]:

$$k_f\left(\frac{\partial T}{\partial y}\right)=\rho_f[\lambda+(c_p)_s(T_m-T_0)]\,v(x,0) \tag{6}$$

in which ρ_f indicates the fluid density and λ the fluid latent heat, T_0 the temperature and $(c_p)_s$ solid surface heat capacity.

By means of Rosseland approximation we can write