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Convection heat and mass transfer of fractional MHD Maxwell fluid in a porous medium with Soret and Dufour effects

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ABSTRACT

This work is concerned with unsteady natural convection heat and mass transfer of a fractional MHD viscoelastic fluid in a porous medium with Soret and Dufour effects. Formulated boundary layer governing equations have coupled mixed time–space fractional derivatives, which are solved by finite difference method combined with L1-algorithm. Results indicate that the Dufour number (*Du*), Eckert number (*Ec*), Soret number (*Sr*) and Schmidt number (*Sc*) have significantly effects on velocity, temperature and concentration fields. With the increase of *Du* (*Sr*), the boundary layer thickness of momentum and thermal (concentration) increase remarkably. The average Nusselt number declines with the increase of *Du* and *Ec*. The average Sherwood number declines with the increase of *Sr*, but increases for larger values of *Sc*. Moreover, the magnetic field slows down the natural convection and reduces the rate of heat and mass transfer. The fractional derivative parameter decelerates the convection flow and enhances the elastic effect of the viscoelastic fluid.

1. Introduction

The natural convection heat and mass transfer in a fluid-saturated porous medium has received considerable attention due to its wide applications, such as geothermal processes, petroleum reservoirs, chemical catalytic reactors, nuclear waste repository, etc. The thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects, which are generally called second order phenomenon of the fluids, become very significant when the temperature and concentration gradients are large. Eckert and Drake [1] presented the importance of these effects in convective transport in mixture between gases with very light and medium molecular weight. Alam and Rahman [2,3] studied numerically Soret and Dufour effects on convection heat and mass transfer flow past a vertical flat plate embedded in a porous medium. Partha et al. [4] considered the effect of double dispersion on free convection from a vertical surface embedded in a non-Darcy porous medium with Soret and Dufour effects. Postelnicu [5] analyzed heat and mass transfer characteristics of natural convection about a vertical surface in porous medium subjected to chemical reaction. Mahdy dealt with the combined Dufour and Soret effects on convection flow embedded in porous medium along a heated vertical wavy surface [6] and from a vertical isothermal plate [7]. Anwar Bég et al. [8] and Hayat et al. [9] examined the Soret and Dufour effects on mixed convection heat and mass transfer from a vertical stretching surface in a Darcian porous medium. Cheng [10] investigated the Soret and Dufour effects on the boundary layer flow due to natural convection over a downward-pointing vertical cone in a porous medium.

More recently, the study of viscoelastic fluids has attracted much interest for its technological applications. Nonlocal fractional model of stress–strain provides a flexible tool for modeling viscoelastic properties: length scale and order of fractional continua [11], which are linked to molecular and system theories [12,13], reflected in the intermittency in the chain segment motions and in the tendency for the particles to cluster and move in a collective fashion. Khan et al. [14,15] dealt with an exact solution for the MHD flow of a generalized Oldroyd-B fluid in porous medium. Hayat [16–18] concerned with deriving the equation for describing the MHD flow of a fractional generalized Burgers' fluid in a porous space. Xue [19] investigated the flow near a wall suddenly set in motion for a fractional generalized Burgers' fluid in a porous half-space. Tripathi and Anwar Bég [20–25] devoted much effort in peristaltic transport of a viscoelastic fluid with the fractional models, which has wide applications in uretral biophysics and potential use in peristaltic pumping of petroleum viscoelastic bio-surfactants. Li et al. studied fractional viscoelastic fluid in por-

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Nomenclature

B_0	applied magnetic flux density
C	(dimensionless) concentration
C_p	specific heat capacity
C_s	concentration susceptibility
C_w	wall concentration
C_∞	ambient concentration
$\overline{C_f}$	average skin friction coefficient
Da	Darcy number
D_m	mass diffusivity
Du	Dufour number
Ec	Eckert number
g	acceleration due to gravity
Gm	modified Grashof number
Gr	Grashof number
K	permeability
k_T	thermal diffusion ratio
L	length of the plate

Greek symbols

α	fractional derivative parameter
α_m	thermal diffusivity
β_C	solutal expansion coefficient
β_T	thermal expansion coefficient
ε	porosity
λ	(dimensionless) relaxation time

Subscripts

w	wall condition
∞	ambient condition
M	magnetic parameter
Nr	buoyancy ratio number
$\overline{Nu}$	average Nusselt number
p	pressure
Pr	Prandtl number
R	Darcy resistance
Sc	Schmidt number
$\overline{Sh}$	average Sherwood number
Sr	Soret number
t	(dimensionless) time
T_m	mean field temperature
T	temperature
T_w	wall temperature
T_∞	ambient temperature
u, v	(dimensionless) velocity components
x, y	(dimensionless) coordinate
μ	dynamic viscosity
v_f	kinematic viscosity
θ	dimensionless temperature
ρ	density
σ	electrical conductivity
f	fluid
m	porous medium

ous medium for helical flows of a heated generalized Oldroyd-B fluid [26] and generalized Maxwell fluid between two infinite parallel plates [27]. Guo and Fu [28] induced the flow of the fractional Jeffreys' fluid by the impulsive motion of a flat plate in a porous half space. Yu et al. [29] obtained the optimal estimation of a Riemann–Liouville fractional derivative for a Stokes' first problem for a heated generalized second grade fluid. Hameed et al. [30] dealt with the peristaltic flow of the fractional second grade fluid confined in a cylindrical tube in the presence of magnetic field and heat source/sink.

However, much of the developments in the theory of fractional viscoelastic fluids in a porous medium have been mainly restricted to exact solutions of the cases when the governing equations are linear. Very little efforts have so far been made to discuss nonlinear convection terms with fractional derivatives. The purpose of the present study is to investigate coupled heat and mass transfer with Soret and Dufour effects by natural convection in a porous medium subjected to applied magnetic field. The fractional Maxwell model and modified Darcy's law are employed to formulate the nonlinear boundary governing equations with mixed time–space fractional derivatives. Numerical solutions are obtained by finite difference method combined with L1-algorithm. The effects of involved parameters on velocity, temperature and concentration fields are presented graphically and analyzed in detail to characterize the complexity of heat and mass transfer of viscoelastic fluid.

2. Mathematical formulation

A two-dimensional unsteady natural convection heat and mass transfer of a viscoelastic incompressible fluid over a vertical plate in a porous medium subjected to magnetic field is considered. The wall is maintained at constant temperature T_w and concentration C_w, while the uniform ambient temperature and concentration far away from the plate are T_∞ and C_∞ respectively. For $T_w > T_\infty$ and $C_w > C_\infty$, an upward flow is induced as a result of the thermal and concentration buoyancy effect so that the Boussinesq approximation is applicable for both the temperature and concentration gradient. The schematic diagram of the problem is shown in Fig. 1, where the x-axis is measured along the plate and y-axis is taken perpendicular to the plate.

It is assumed that: (i) the fluid and the porous medium are in local thermodynamic equilibrium; (ii) the porous medium is isotropic and homogeneous; (iii) the properties of the fluid and porous medium are constants except that the influence of density variation with temperature has been considered only in the body-force term; (iv) there is no applied voltage and the magnetic Reynolds number is small, hence the induced magnetic field and Hall effects are negligible. Under these assumptions, the momentum equation in a porous medium is given by:

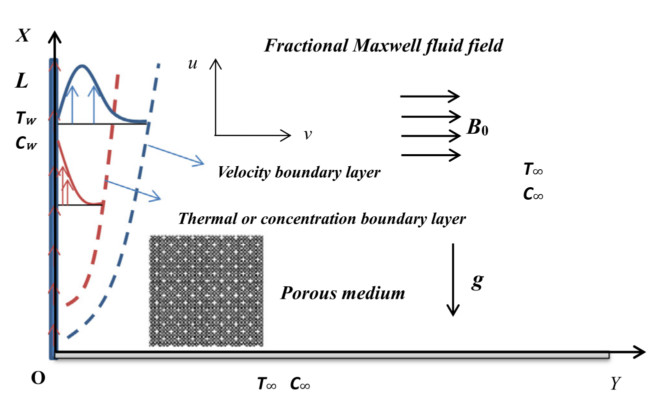

Fig. 1. Schematic diagram of the physical system.