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Design explorations of heat conductive pathways

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ABSTRACT

Constructal theory, now called constructal law, has influenced and driven a significant amount of research and attention over the past 2 decades due to its significance in understanding designs found in nature. It is worth to remember that the origins of constructal law can be traced back to the design of heat conduction paths, alternatively called access problem, in Bejan's 1996 paper. However, it is evident that although branching designs are generated, geometric and 'constructal' constraints are always present and often not relaxed in their designs. In this paper, additional design explorations to solutions for the access problem are sought and discussed. Density based topology optimization under 6 different, tunable material interpolation models are chosen since design explorations can be considered to be straightforward for finding solutions and the design process is relaxed. The variation in the material property model highlights the importance of search spaces in any design process. To give fair comparisons, the final designs obtained from the optimization process are projected as 0–1 designs. The results reveal other topologies that perform better as compared to two constructal design cases. The significant difference starting from the fundamentals of constructal designs is also discussed.

1. Introduction

We would like to start off with a restatement of Bejan's [1] introduction in his seminal paper for constructal theory in 1996 (clarifications on the publication date is given in [2]):

> "Some fundamental problems appear 'obvious', but solutions can only be realized after considerable technological progress has been made..."

This statement was originally intended to point out the problem for better electronics cooling and packaging, which has continued to be one of the main driving areas for research in heat transfer today. This statement also holds true for topology optimization that has proven to be a useful design tool in solving fundamental problems related to material lay-out and shape determination. On another point of view, topology optimization itself has relied (and is continuing to rely) on the progress of computing technologies before milestones on both methods and problem scales are realized.

It can also be claimed that the 'true optimal solution' for the cooling problem is still yet to be found on the basis of material technology and science, design and manufacturing constraints, uncertainty and reliability of both operating space and design spaces, and computational exploration to name a few. Continuous research efforts on design methods, materials and manufacturing technologies are constantly striving to find better solutions.

Nature, on the other hand, has had the time to continuously improve and adapt. These improvements and adaptations are present in a wide variety of designs that we observe in everyday life. Thus, designs that are 'nature-inspired' have been given more attention.

The formal statement of the constructal law [3] is given as the following:

> "For a finite-size flow system to persist in time (to live) it must evolve such that it provides greater and greater access to the currents that flow through it."

This formal statement had also originated in the constructal theory paper for heat conducting paths [1] which was addressed to solve the access problem in the context of heat transfer. Its philosophical contribution, however, has begun almost simultaneously in street networks [4]. Its conceptual integration into two textbooks [5,6] that covered direct significance to thermodynamics (consequently covering heat transfer) and nature is attributed to its wide growth in the literature. It has then continued to span a much greater context, see [3,7–10] to name a few. Through the

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constructal law, the interconnectivity of both the tangible and intangible designs can be explained and how they are approaching designs found in nature. It was also highlighted in [11] that the constructal theory was first proposed to expand thermodynamics in a fundamental way, and that there is a phenomenon in the generation of configuration. In this paper, configuration is broadly put as topology.

In this paper we would like to revisit the access problem in the perspective of topology optimization with comparison to the original constructal topology. Explorations on a wide variety of both better and worse designs, when compared to the constructal "T" shaped geometry, are made. We emphasize search spaces, in the form of the different material interpolation models, which leads to these designs.

2. The access problem

The access problem, considered in this paper, is interpreted slightly different compared to the original paper of constructal theory. In the original paper, material distribution is treated as heat transfer inserts. This in turn allows treatments for the inserts as slender fins. In this paper, the distribution is done purely for a two dimensional problem and no slenderness nor geometric constraints are considered. The problem is formally stated as:

> "Consider a discrete and finite-sized domain, Ω, in which heat is being generated at every discretization, $d\Omega$. It is cooled through a small patch (heat sink) located on its boundary with a known temperature, T_0. A finite volume, V_{mat}, of high conductivity material, $k_{\max}$ is available. If the high conductivity material is not present, a low conductivity material, $k_{\min}$, will be in its place. *The presence or absence of* $k_{\max}$ *will not affect the heat being generated in the domain.* Under steady state conditions, determine the distribution of $k_{\max}$, under the constraint of V_{mat}, through which a thermal performance function, c, is minimized."

The italics in the problem statement is a clarification due to the different interpretations and variation of some heat conduction problems over the years. The problem is also posed in such a way that heat is still being generated on the discretized domain even when $k_{\max}$ is present. Several literature had treated this problem differently like Mathieu-Potvin and Gosselin [12] which had assumed on one of the element types that if material is present, no heat will be generated on the discretized domain or like Kobayashi et al. [13] who have assumed that the walls of the structure could be treated as an isothermal surface. These differences in treatment for the behavior of the domain having $k_{\max}$ cannot be judged as correct or incorrect since these are their design assumptions and they are solving different problems all related to the distribution of $k_{\max}$. A visualization of these deviations are presented in Fig. 1. It is again re-iterated that this paper is finding solutions for a 2D case in Fig. 1(a).

The original problem in [1], as shown in the left hand side of Fig. 1(a), was intended for heat conductive inserts. Several instances in which heat conductive paths were also investigated are: bionic optimization by Cheng et al. [14], combinatorial approaches by Xu et al. [15], evolutionary algorithms by Mathieu-Potvin and Gosselin [12], and density based methods with Solid Isotropic Material with Penalization (SIMP) interpolation [16,17] to name a few. It was recently raised by Bejan [18] that both [14] and [15] had conceptually similar contributions to [19] but termed and placed in a different context. On the other hand, topology optimization has been recently used for the layout of heat sinks or heat dissipation devices as demonstrated in [20]. In earlier years of topology optimization, most of the papers that have treated the problem as a heat sink problem, have not considered convective effects in the convective boundaries of the heat sink due to the additional complexity it brings. In this paper, the same treatment is applied for the given examples. Recent works during the past decade has included considerations of both forced convection [21] and natural convection models [22]. A complete product development cycle of a 3D printed heat sink which considers forced convection effects was also recently published by Dede et al. [23]. Their experimental results in [23] were not in good agreement with their simulations primarily due to the manufacturing process and the final effective material properties of the heat sink. Embedded flow structures, as shown in Fig. 1(b), was considered as an assumption in a more recent Constructal Law paper [24]. In their assumptions, it was said boundaries are modeled as isothermal cylinders, and this approximation was good for investigating configuration effects for pipes containing heat-pump fluids buried in soil. In topology optimization, more rigorous investigation consisting of multi-physics models (heat conduction and fluid flow) have been investigated. An example worth mentioning are the works of Koga et al. [25], who has demonstrated the full product development cycle of a water-cooled heat sink device. Their experiments have also matched their simulation results with high accuracy. Comparisons between a commercially available, conventional pin-fin heat sink has also shown that the thermo-hydraulic performance of the optimized heat sink is better [26].

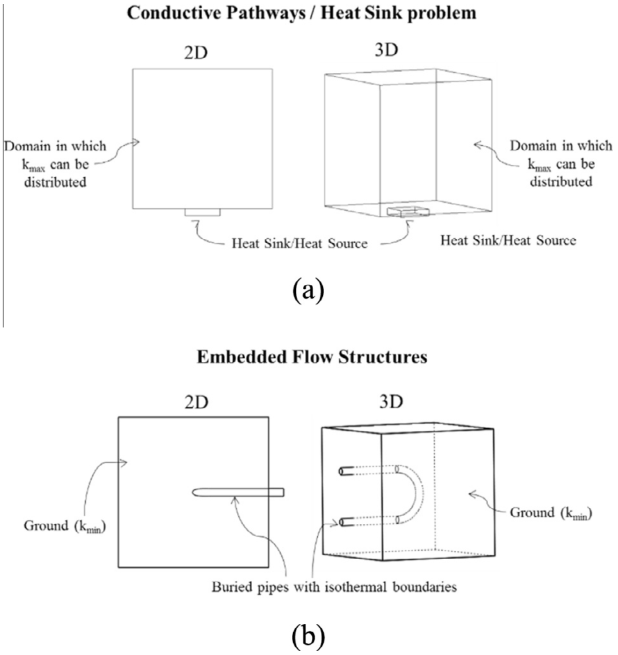

Fig. 1. Some problem variations in heat conduction problems over the years.

In the defined problem, a discretization of the finite sized domain is considered as the smallest system. In constructal theory, the problem is treated as geometric optimization (a sizing problem) of basic elemental volumes, with $k_{\max}$ material conductivity. Constraints were also posed in the basic elemental volumes to be of slender rectangular area. Constraints in the assembly of the basic elemental volumes were also constrained in the original paper to be of right angles (T-shaped). The assembly constraints were recently relaxed and re-explored (Y-shaped) [24]. Best performance was achieved when every angle was optimized at every level of assembly. In most FEA based topology optimization, the discretization are represented by elements. For simplicity, elements in topology optimization papers are usually represented as equally sized quadrilateral elements for 2D, and equally sized