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Magnetohydrodynamic effects on peristaltic flow of hyperbolic tangent nanofluid with slip conditions and Joule heating in an inclined channel

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ABSTRACT

This investigation looks at the influence of an inclined magnetic field on peristaltic transport of hyperbolic tangent nanofluid in inclined channel having flexible walls. Nanofluid consisting of Brownian motion and thermophoresis effects is employed in the definition of problem. Thermal radiation and Joule heating are present. Formulation is further completed by consideration of slip conditions in terms of velocity, temperature and concentration. Lubrication approach has been followed for the development of problem formulation. The key role of various involved parameters on the flow phenomenon are sketched. Slip effect causes the velocity and temperature to increase while reduces the concentration. Reduction in temperature is noticed with thermal radiation whereas temperature enhances for larger Hartman number in responce to the Joule heating effect.

1. Introduction

The mechanism by which contents of the food moves ahead under the influence of progressive wave of contraction and expansion is termed as peristalsis. In particular this activity involves mixing and pushing materials through contraction or expansion of the waves propagating along the channel walls. The phenomenon has key role in many physiological processes like urine transport from kidney to bladder through the ureter, transport of lymph in the vessels, swallowing food through the esophagus, the movement of chyme in the gastrointestinal tract, ovum movement in the fallopian tube, movement of spermatozoa in the cervical canal and bile movement in a bile duct. This mechanism is also influential with reference to chemical processes and medical industry which covers heart lung machine, noxious fluid transport, roller and finger pumps, novel pharmacological delivery systems and locomotion of worms etc. An extensive information on the topic existed in literature after the work initiated by Latham [1] and Shapiro et al. [2] via theoretical and experimental approaches. Apart from their works latest studies in this area can be seen through the Refs. [3–8].

At present the mathematicians, modelers, computer scientists, engineers and physiologists seem to have interest in the research area of nanofluids related to peristaltic activity. Nanofluid is a traditional liquid with nanosize particles. Utilization of nanofluids has become more important due to its involvement in surgery, vivo therapy, protein engineering, cancer diagnosis and therapy, drug delivery, neuro electronic interfaces, photodynamic therapy, nonporous materials for size exclusion chromatography, shedding new light on cells, molecular motors like kinesis and charge based filtration in the kidney basal membrane etc. Even some scientists thought of using nanofoods to trick the body into feeling fuller for longer thus stopping overeating. The word "nanofluid" was coined by Choi [9]. Thus under different assumptions the works on nanofluids have been progressed by many researchers (see Refs. [10–15] and the relevant studies therein). Also importance of nanofluids in peristalsis has not been ignored by the scientists which can be witnessed through research articles (see Refs. [16–19]). Further the idea of magnetohydrodynamics is remarkable in high temperature equipments such as power generators and in conductive physiological materials including the blood, blood pump machines, Magnetic Resonance Imaging (MRI), magnetotherapy, hyperthermia, arterial flows and in the phenomenon of aviation used by an airplane's compass where the influence of inclined magnetic field is found meaningful. Some useful studies in this area can be cited through Refs. [20–22]. Moreover the flexibility of the walls is essential for the case when gap between the channel boundaries is small. Thus Hayat et al. [23,24] explored MHD peristaltic transport of Jeffrey fluid in a symmetric and

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rotating channels. Reddy and Reddy [25] studied influence of Joule heating on MHD peristaltic flow of a nanofluid in compliant walls channel. Study of peristalsis under slip effects,chemical reaction and compliant walls have been examined in the past (see Refs. [26,27]). Hayat et al. [28] studied effects of an inclined magnetic field on peristaltic flow of Williamson fluid in an inclined channel with convective conditions.

Hyperbolic tangent fluid predicts shear thinning behavior i.e, its viscosity drops upon enhancement in the shear rate. Lava, ketchup, paint and whipped cream are few examples of such fluids. It is seen that not much has been said about peristalsis of hyperbolic tangent fluid. Only few works in this direction have been presented (see Refs. [29–31]). Hence considering all such facts in mind, the current work addresses the impact of inclined magnetic field on peristaltic flow of hyperbolic tangent nanofluid in an inclined channel with flexible boundaries. In addition thermal radiation effects along with Joule heating are retained in the energy equation. Such consideration is important in natural mechanism driven under the impact of sharp magnetic field like in human body where the influence of flexible walls are also dominant. The channel walls exhibits velocity, thermal and concentration slip conditions. Graphical illustrations have been obtained through numerical solution for the velocity, temperature, concentration and heat transfer coefficient. Effects of sundry parameters on the physical quantities of interest are explained in detail.

2. Modeling

An incompressible hyperbolic tangent nanofluid in an inclined channel of uniform thickness $2d_1$ is considered (see Fig. 1). Channel walls are taken flexible. The axial and transverse directions are indicated by x and y respectively. An inclined magnetic field of strength B_0 has the following form

$$\mathbf{B} = (B_0 \sin\beta^*, B_0 \cos\beta^*, 0). \tag{1}$$

Here β^* and ζ^* are the inclinations of magnetic field and channel respectively. A sinusoidal wave of wavelength λ and constant wave speed c propagates along the channel boundaries. Brownian motion effects along with thermophoresis are retained here. Further thermal radiation is present in the problem. The role of electric and induced magnetic fields is considered negligible. The wall geometry can be described by the expression

$$y = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right], \tag{2}$$

where a is the wave amplitude and t the time.

Expression of Cauchy stress tensor $\boldsymbol{\tau}$ for hyperbolic tangent fluid is [30]:

$$\boldsymbol{\tau} = -p\mathbf{I} + \mathbf{S}, \tag{3}$$

$$\mathbf{S} = \left[\mu_\infty + (\mu_0 + \mu_\infty)\tanh{(\Gamma\dot{\gamma})^m}\right]\dot{\gamma}. \tag{4}$$

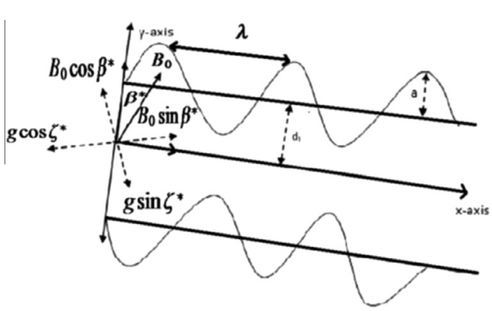

Fig. 1. Geometry of the problem.

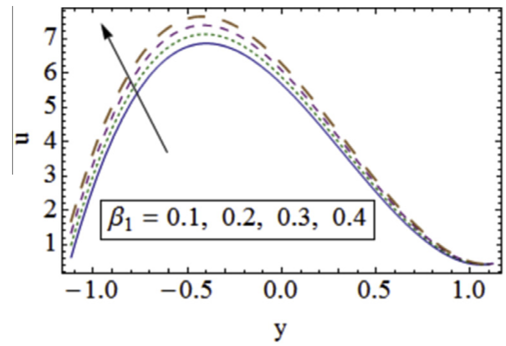

Fig. 2. Sketch of u for β_1 when $E_1 = E_3 = 0.01, E_2 = 0.02, Rn = 0.5, \beta_2 = \beta_3 = 0.1, m = 0.1, \Pr = 0.8, Nt = 0.2, Nb = 0.3, Fr = 1.5, \mathrm{Re} = 0.8, Br = 1.7, We = 0.01, M = 1, \zeta^* = \beta^* = \frac{\pi}{3}$.

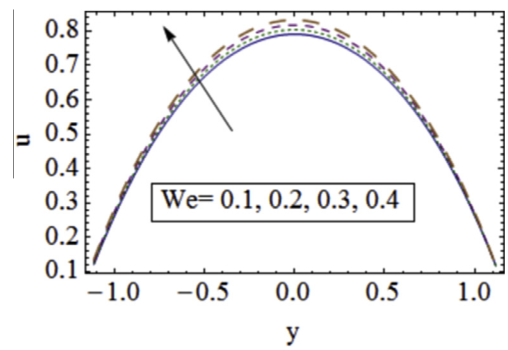

Fig. 3. Sketch of u for We when $\beta_2 = 0.1, \beta_3 = 0.1, \beta_1 = 0.2, Rn = 1.5, m = 0.11, \Pr = 1, \mathrm{Re} = 0.8, M = 2$.

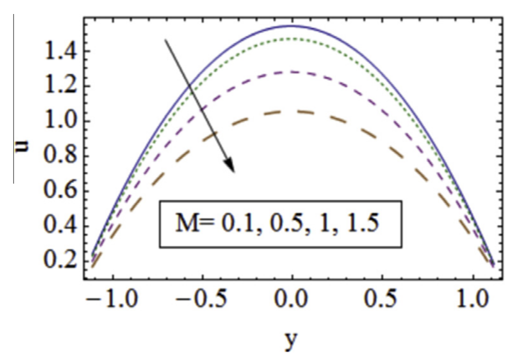

Fig. 4. Sketch of u for M when $m = 0.1, \Pr = 1.5, Br = 1.7, \mathrm{Re} = 0.8, We = 0.5, \zeta^* = \beta^* = \frac{\pi}{4}$.

Here $\mathbf{S}$ represents an extra stress tensor, Γ the time dependent material constant, μ_∞ the infinite shear rate viscosity, μ_0 the zero shear rate viscosity, m the power law index, p the pressure, $\mathbf{I}$ the identity tensor and $\dot{\gamma}$ is defined by [29–31]:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi}, \tag{5}$$

where $\pi = \frac{1}{2}tr\left(\mathrm{grad}\mathbf{V} + (\mathrm{grad}\mathbf{V})^T\right)^2$ and $\mathbf{V} = (u, v)$ the velocity components in the axial and transverse directions respectively. Considering $\mu_\infty = 0$ and shear thinning effects of hyperbolic tangent fluid i.e $\Gamma\dot{\gamma} \prec 1$ one obtains

$$\mathbf{S} = \mu_0\left[(\Gamma\dot{\gamma})^m\right]\dot{\boldsymbol{\gamma}} = \mu_0\left[(1+\Gamma\dot{\gamma}-1)^m\right]\dot{\boldsymbol{\gamma}} = \mu_0[1 + m(\Gamma\dot{\gamma}-1)]\dot{\gamma}. \tag{6}$$