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Three-dimensional numerical investigation on thermosolutal convection of power-law fluids in anisotropic porous media

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ABSTRACT

The present study simulates the 3D unsteady double-diffusive natural convection subject to opposing thermal and solutal buoyancy forces ($N < 0$) in a porous cubic by a generalized non-Darcy model, in which the effects of the crucial parameters such as the porous thermal Rayleigh numbers, buoyancy ratio and anisotropy ratio on the flow structure, heat and mass transfer of power-law fluids are investigated independently. The top and bottom walls are given different temperatures and concentrations, while the other walls are adiabatic and impermeable. A compact high order finite volume method is adopted to describe the flow structure and the resulting heat and mass transfer characteristic of non-Newtonian fluids in the anisotropy porous cubic. Our simulations show that the flow structure develops from conduction-dominated to convection-dominated as buoyancy ratio or anisotropy ratio or porous thermal Rayleigh number increases. The average Nusselt and Sherwood numbers keep constants during the conduction-dominated stage, then increase along the transition route. On the other hand, the impacts of different power-law indexes on the convection are mainly manifested in rheological properties, which elucidate that the shear-thinning fluids is more effective in heat and mass transfer enhancement than shear-thickening fluids. The studies may help us establish a physically reasonable methodology to systemically assess double-diffusive convection of non-Newtonian power-law fluids in anisotropy porous media in the real world.

1. Introduction

Thermosolutal or double-diffusive convection, which is generated by buoyancy due to combination of temperature and concentration gradients, is relevant to a number of industrial applications such as the contaminant transport in saturated soils, the underground disposal of nuclear waste, petroleum drilling, chemical and food processing, etc. Over the past decades, studies on this subject have been reflected by a number of publications [1–10]. The 3D double-diffusive natural convection in a solar distiller was numerically studied by Ghachem et al. [11]. They found the effects of buoyancy ratio on the flow structure and entropy generation at a fixed $\text{Ra} = 10^5$ and expanded the results to the magnetic field [12]. Chen et al. [13] considered a large parameter range to study the bifurcations in 3D double-diffusive convection. Diersch et al. [14] utilized the finite-element method to analyze the 3D double-diffusive convection of groundwater flow (saturated porous medium) and a 3D Benard convection process. Remarkable works about the 3D convection did by Sezai et al. [15–17] are worthy of attentions. They [15] proposed a 3D Brinkman extended Darcy model to study double-diffusive convection in a fluid-saturated porous cubic enclosure. Also, the flow bifurcations of a 3D Rayleigh–Benard convection in an investigated cavity was investigated [16]. Then, they [17] studied the flow transitions in 3D double-diffusive fingering convection in a porous cavity. Recently, Hadidi et al. [18] reported a numerical study about the 3D double-diffusive natural convection across a cubical enclosure partially filled by vertical porous layer.

Above investigations discussed so far refer to Newtonian fluid flow. In industrial practices, however, it is of great importance to study double-diffusive convection of non-Newtonian fluids. Both Benhadji et al. [19] and Khelifa et al. [20] used the Darcy model to analytically and numerically study the double-diffusive convection in 2D shallow porous cavity filled with power-law fluids, while the latter [20] further considered the Soret effect on buoyancy. Jena et al. [21] investigated double-diffusive free convection of a non-Newtonian Ostwald–De Waele fluid inside a 2D cavity having partially-active vertical walls. Kefayati [22,23] did a series

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Nomenclature

C	concentration
c_p	heat capacity of porous medium
c_f	heat capacity of fluid
Da	Darcy number
$\overline{\overline{D}}$	second-order tensor of mass diffusivity
D_x	mass diffusivity in x-direction
D_y	mass diffusivity in y-direction
D_z	mass diffusivity in z-direction
g	gravitational acceleration
$\overline{\overline{K}}$	second-order tensor of permeability
K_x	permeability in x-direction
K_y	permeability in y-direction
K_z	permeability in z-direction
$\overline{\overline{k}}$	second-order tensor of thermal diffusivity
k_x	thermal diffusivity in x-direction
k_y	thermal diffusivity in y-direction
k_z	thermal diffusivity in z-direction
k_p	thermal conductivity of porous medium
k_f	thermal conductivity of fluid
Le	Lewis number
n	power-law index
Nu	Nusselt number
N	buoyancy ratio
p, P	pressure
Pr	Prandtl number
$\overline{Q}$	vertical unit vector
Ra	porous Rayleigh number
Ra_T	thermal Reyleigh number
Ra_S	solutal Reyleigh number
Sh	Sherwood number
T	temperature
t	time
$\mathbf{u}$	velocity vector
u, U	velocity in x direction
v, V	velocity in y direction
w, W	velocity in z direction
x, X	x-coordinate
y, Y	y-coordinate
z, Z	z-coordinate

Greek symbols

γ	heat capacity ratio
ρ	density
Θ	dimensionless temperature
Φ	dimensionless concentration
ϕ	porosity of the porous media
α	thermal diffusion coefficient
β_T	thermal expansion coefficient
β_C	concentration expansion coefficient
μ, $\bar{\mu}$	dynamic viscosity
ω_1	rotating angle in the x-axis
ω_2	rotating angle in the y-axis
ω_3	rotating angle in the z-axis
θ	anisotropy ratio
τ_{ij}	shear stree tensor
τ	dimensionless time
$\boldsymbol{\tau}$	shear stress tensor

work to study the entropy generation and magnetic field effect in double-diffusive convection of power-law fluids in a 2D cavity with or without the Soret and Dufour effects. Lack of discussion on the double-diffusive convection of power-law fluids in three-dimension may sometimes decrease the evaluation of the subject in this regard.

Meanwhile, most of the previous studies were concerned with a variety of the homogenous porous media subject to fluxes of heat and mass applied in the same direction. In fact, the anisotropy, which is the consequence of asymmetric geometry of the grain or fibers, is encountered commonly in nature such as rock formations. Recently, the effects of anisotropy in porous media have attracted the attention of many researchers. Among the early works, Royer et al. [24] developed a simple dimensionless expression for the 2D natural convection heat transfer in an anisotropic and heterogeneous porous medium with internal heat generation. Through the generalized non-Darcy approach, Krishna et al. [25] numerically analyzed the natural convection in a 2D anisotropic porous cavity with a finite heat source at the bottom wall, and Nithiarasu et al. [26] studied the natural convective flow in a hydrodynamically and thermally anisotropic porous medium. Nield et al. [27] investigated the effect of strong heterogeneity on the onset of convection induced by a vertical density gradient in a saturated porous medium governed by Darcy's law. Also, interest regarding double-diffusive flow has surged in view of its applications in various engineering problems. Bera et al. [28] studied the multi-solutions and oscillations of double-diffusive convection in an anisotropic porous cavity using the Darcy model. Oueslati et al. [29] presented a numerical study of thermosolutal convection in a 2D anisotropic porous cavity subject to cross-fluxes of heat and mass. Khadiri et al. [30] studied numerically the 2D and 3D multiple steady states of double-diffusive convection in homogeneous and isotropic porous media.

Presently, however, the 3D double-diffusive convection of non-Newtonian power-law fluids in the anisotropic porous media has not been reported enough. In this paper, we attempt to propose an initial numerical study the 3D convection of non-Newtonian fluids in the general anisotropic porous media. Hence, the effects of the buoyancy ratio, anisotropy ratio, power-law index and porous thermal Rayleigh number on the 3D aspects, heat and mass transfer of power-law fluids are exactly the focus of the present work. In the calculation, the third-order Runge–Kutta and the compact high order finite volume methods are used to promote the simulation.

2. Mathematical model

The physical model shown in Fig. 1 consists of a porous cubic cavity. The domain, which is anisotropic in permeability, is filled with the non-Newtonian fluids. The bottom wall is considered to be maintained at high temperature T_h and high concentration C_h, while the top wall is at a lower temperature T_l and lower concentration C_l. The vertical walls are adiabatic and impermeable.

To derive the governing equations, some important assumptions are clarified as follows:

- The thermosolutal convection in the porous cubic cavity is laminar, incompressible and unsteady.
- The Ostwald–De Waele (power-law) model is taken for the non-Newtonian fluid.
- The solid matrix of the porous media does not undergo deformation and is anisotropic in permeability, thermal diffusivity and mass diffusivity, which are defined as the second-order tensor $\overline{\overline{K}}$, $\overline{\overline{k}}$ and $\overline{\overline{D}}$, respectively [28].
- The fluid is in local thermal equilibrium with the solid matrix.