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Stability of homogeneous seepage of a liquid mixture through a closed region of the saturated porous medium in the presence of the solute immobilization

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ABSTRACT

The linear stability problem of homogeneous vertical seepage of the liquid mixture through a closed region of porous medium is investigated. The problem is solved taking into account immobilization of the solute by the porous medium, which is described within the MIM approach based on the second-order kinetics model allowing for the saturation of the solid matrix of porous medium with immobilized solute. The stability of the homogeneous vertical seepage mode to two-dimensional perturbations has been considered for the case of square porous domain. It is assumed that the lower and upper boundaries are maintained at fixed different concentrations of solute and the lateral boundaries are impermeable to the mixture. It has been found that due to consideration for the solute immobilization the perturbation spectrum of the homogeneous vertical seepage mode exhibits oscillatory perturbations which are absent in the case of using the classical diffusion model. Neutral curves for the space of the respective parameters and plots of neutral perturbation frequency as the function of governing parameters have been obtained numerically. It has been shown that in the presence of the immobilization the stability of the examined seepage mode decreases as compared to the classical diffusion model. The numerical investigation has shown that the oscillatory mode of convection exists only in a certain range of parameters characterizing the intensity of sorption. The maps of oscillatory instability mode existence in the space of governing parameters have been obtained.

1. Introduction

Porous media have a rather complicated spatial structure. The drift of particles in such a medium is not always governed by the Gaussian distribution law since the solute particles can stick to a solid matrix (immobilized), which slows down the solute transport through the porous medium.

The particles adhesion to a solid matrix (the adsorption effect) occurs continuously all over the volume. For small particles such as two- and triatomic molecules of solute the effect is of minor importance. However, large enough particles can be immobilized more actively, which essentially slows down the diffusion [1,2].

In the present work, immobilization is described within the MIM (mobile-immobile medium) model with nonlinear sorption kinetics taking into account the saturation of the solid matrix of porous medium with solutes [3].

The MIM approach is based on the assumption that there are two phases of solute: immobile (adsorbed or trapped solutes) and mobile (drifting with the flow). The evolution of mobile phase concentration is described by the classical diffusion equation including the term, which describes the transfer of solute into the immobile phase. The dependence of the rate of solute transfer into the immobile phase on the solute concentrations in two phases describes the kinetics of the "phase transition".

The model of this kind was first proposed in [4] and a few months later in [5]. In these studies the linear relationship $c = \xi q$ between the concentrations of immobilized (q) and free (c) solutes is taken as a kinetic law. Such an approach allows us to describe the slowing of the diffusion process, but leads to a decrease in the diffusion coefficient by $(1 + \xi)$ times. The proposed relationship gives a poor fit to the experimental data, as was noticed in [6,7].

Later on, few attempts have been made to construct models using the Freundlich and Langmuir adsorption isotherms, which describe rather well the equilibrium state in the case of adsorption of gases and fluids from solutions of low concentration [8–11]. These models

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describe the diffusion better than the models in[4,5], although the isotherms themselves correspond to a steady-state regime (characterized by dynamic equilibrium between the phases).

To describe the dynamics of the immobilized solute, it is necessary to use some kinetic equation. The simplest model of such kind is the first-order kinetics model proposed in [6] and developed in [7], in which the immobilization rate linearly increases with increasing concentration of the mobile solute and linearly decreases with increasing concentration of the immobile solute. However, this model also shows a poor fit to experimental data, which is due to the fact that it does not take into account the effect of saturation of the immobile phase (that is, the concentration of immobilized solute should not exceed some limiting value). This effect has been first considered in [3] within the model with the second order kinetics, in which the adsorption coefficient linearly depends on the difference between the limiting saturation concentration of porous medium and concentration of immobile solute. This model allows one to adequately describe the experimental data over a wide range of solute concentrations.

More complicated situations requiring consideration of medium heterogeneity or some of its geometrical and chemical properties are generally described in the framework of higher-order models (see, for example, [12]). The models of the described type can also be applied to the case when the contact of the solute with solid matrix is accompanied by chemical reaction. For instance, to take into account the reaction of solute decomposition (sedimentation on the walls in the form of insoluble phase) the kinetic equation, in which the adsorption rate is independent of the concentration of immobilized solute can be used. The simplest equation of this kind was used in paper [13]. Another kind of MIM-type models is fractional MIM model [14]. This type of MIM model is used for description of the effects at the small solute concentrations when the dependence of solute concentration on time is represented by power law. Some problems where this effect are important is described in [15–19].

In our work, we study the stability of vertical seepage of a liquid mixture through a closed porous domain. The concentration difference prescribed between the upper and lower boundaries initiates a convective motion in the gravity field. The development of convection in a closed volume of porous medium was studied for the first time in [20] where the conditions for mechanical equilibrium are obtained and its stability is investigated. It is found that the lower level of instability is doubly degenerated. The experiments described in [21] supported the validity of the conclusions drawn in [20]. In [22] it was demonstrated that the double degeneration revealed in [20] is the result of the equation co-symmetry property.

In more recent studies [23–26] the conditions for the onset convection in a porous medium was investigated numerically and analytically for the domains of different configuration.

The dynamic properties of two-dimensional convection in a porous medium in case of weak violation of the conditions provoking degeneration was investigated in [27,28]. It was found out that weak deviations from the conditions described in [20] eliminate degeneracy. The phase diagrams were constructed and the mechanism of degeneracy elimination was analyzed. It was shown that in the case of weak vertical homogeneous seepage of the fluid through the porous domain a monotonic instability mode is realized.

In paper [29], the authors investigated the stability of vertical homogeneous seepage of the fluid through the porous layer heated from below, the basic state is defined and the critical parameters are evaluated. The studies made in [30,31] have shown that in the case when the symmetric boundary conditions are applied to the upper and lower boundaries (both are perfectly conductive and impermeable as in our case) the seepage enhances stability as compared to the case where the seepage is absent.

In most works, the solute immobilization is not taken into account. Indeed, it does not make any influence on steady states or monotonic instability. However, in the case of unsteady base states or oscillatory instabilities, as shown in [19,32], the immobilization leads to the stabilization of the system since some part of solute becomes immobile. As it is shown in the present paper, immobilization can also result in the appearance of new oscillatory regimes.

The objective of the present paper is to investigate the influence of solute immobilization on the stability of homogeneous vertical seepage of mixture through a closed porous domain. The paper comprises four sections. In the first Section, the constitutive equations are derived within the MIM model to describe the solutal convection in a porous medium accounting for solute immobilization. The second Section contains the statement of the problem, the solution for the basic state and the derivation of equations for the perturbations. In the third Section the results of stability analysis are discussed. The fourth Section summarizes the main findings of the present study.

2. Problem formulation. Governing equations and boundary conditions

The two-dimensional flow of liquid mixture in a square porous domain is considered. Solute concentrations at the upper and lower boundaries of the domain are kept constant and denoted by C_+ (upper) and C_- (lower) such that $C_- < C_+$ (see Fig. 1). The lateral walls of the domain are assumed to be impermeable. The seepage of the mixture through this domain occurs in the direction of the gravity field, i.e. the system works as a filter providing strictly vertical filtration. The boundary condition of fixed concentration are more difficult for experimental realization than the conventionally used condition of mass flux vanishing, but with such condition the problem solution is substantially simplified and fundamental features of the solution will be the same in both cases.

The described problem can be written within the Darcy–Boussinesq model with account of solute immobilization as [7,31]:

$$\begin{aligned}
&\phi \tfrac{\partial}{\partial t}(c+q) = -\mathbf{V}\cdot\nabla c + \phi D \nabla^2 c,\\
&\tfrac{\partial q}{\partial t} = \mathcal{R}(c, q),\\
&-\nabla p = \tfrac{\eta}{\kappa}\mathbf{V} + \rho_l \beta_c c g \mathbf{j},\\
&\nabla\cdot\mathbf{V} = 0, \quad p = p' + \rho_l g z,\\
&\mathbf{V}_\perp|_{y=0,L} = V_0, \quad \mathbf{V}_\perp|_{x=0,L} = 0,\\
&\tfrac{\partial}{\partial x} c\big|_{x=0,L} = 0,\; c|_{y=0} = C_+, \quad c|_{y=L} = C_-.
\end{aligned} \tag{1}$$

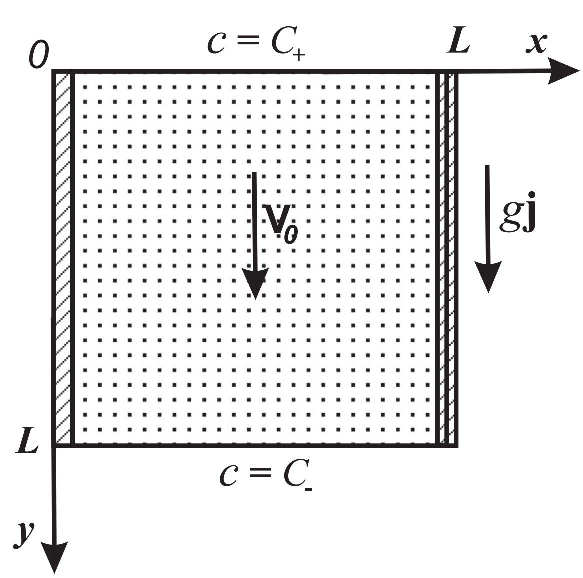

Fig. 1. Problem configuration.