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Unified Nusselt- and Sherwood-number correlations in axisymmetric finite-gap stagnation and rotating-disk flows

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ABSTRACT

This paper develops a unified analysis of stagnation flow heat and mass transport, considering both semi-infinite domains and finite gaps, with and without rotation of the stagnation surface. An important objective is to derive Nusselt- and Sherwood-number correlations that represent heat and mass transport at the stagnation surface. The approach is based on computationally solving the governing conservation equations in similarity form as a boundary-value problem. The formulation considers ideal gases and incompressible fluids. The correlated results depend on fluid properties in terms of Prandtl, Schmidt, and Damköhler numbers. Heterogeneous chemistry at the stagnation surface is represented as a single first-order reaction. A composite Reynolds number represents the combination of stagnation flows with and without stagnation-surface rotation.

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1. Introduction

Axisymmetric stagnation flows have the remarkable property that heat- and mass-transfer fluxes to the stagnation surface can be highly uniform. This characteristic is technologically valuable and reactors are designed to exploit the inherent flux uniformity. Implementations of stagnation flows find numerous applications in technology, for example chemical-vapor-deposition (CVD) reactors [1–5] and rotating-disk electrodes [6–8], as well as in laboratory-scale reactors that support fundamental research into reaction chemistry in such areas as materials deposition, electrochemistry, catalysis and combustion [9–21].

The objective of the present paper is to develop general correlations for heat and mass transfer at the stagnation surface. In dimensionless terms, this means developing correlations for the Nusselt and Sherwood numbers in terms of their dependence on the Reynolds, Prandtl, Schmidt, and Damköhler numbers. Achieving such general correlations depends upon significant assumptions and simplifications, including constant properties and single-step chemistry.

The correlated results provide valuable insights and rules of thumb for scaling relationships and expected reactor behavior. For example, knowledge of the heat transfer coefficient (Nusselt number) is useful in reactor design to determine the required power to a heated stagnation surface as a function of gas flows, pressure, etc. in a deposition system. Understanding of the mass-transfer coefficient (Sherwood number) can provide guidance in operating-parameter trade-offs when optimizing deposited-material quality, which might include temperature or pressure dependence or minimizing parasitic chemistry. Another design consideration might be understanding the effects of actively heating or cooling the reactor inlet manifold on expected growth rate. However, such correlations certainly do not replace more comprehensive computational fluid dynamics (CFD) models for detailed reactor and process design and development [22–25].

1.1. Background

The basic mathematical behavior of stagnation flow were first recognized and reported by Heimenz in 1911 [26]. In 1936, Homann extended Heimenz' planar analysis to include axisymmetric flow [27]. In 1921, von Kármán reported a mathematically analogous flow in the boundary layer above a rotating disk [28]. These early research initiatives were based on physical and mathematical insight and analysis, without the benefit of computational solution. Both the stagnation and rotating-disk flow analyses led to combining the continuity and momentum equations into a single third-order ordinary-differential-equation boundary-value problem. Although these early papers are historically consequential, they are practically inaccessible for many readers. The fundamental theory, derivations, and analyses may be found in more recent

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texts and monographs on fluid mechanics and boundary layers [29–31,30,32]. The mathematical and computational development in the present paper follows most closely that reported by Kee et al. [29].

There is a vast literature concerning stagnation flows and rotating-disk flows. Historically, these flows have been recognized as being closely related, but have been modeled as being distinctly different. In fact, as discussed by Kee et al. [29], the stagnation and rotating-disk flows, including the semi-infinite and finite-gap domains, can be represented with a common set of differential equations.

The historical stagnation-flow literature is based upon assuming a semi-infinite environment above the viscous boundary layer near the stagnation surface, where the outer flow behaves as an inviscid potential flow. Although this formulation is appropriate in some settings (e.g., external aerodynamics), it is not generally appropriate for confined-flow applications such as CVD reactors or condensed-phase vessels. As illustrated in Fig. 1, the feed stream may be introduced at a specified flow rate through a manifold a some fixed height above the stagnation surface. This so-called finite-gap setting changes the mathematical characteristics of the boundary-value problem to be solved [29,33–38]. Specifically, the finite-gap formulation requires the introduction of an eigenvalue associated with the radial pressure gradient.

As illustrated by the streamlines in Fig. 1 the velocity field clearly has two-dimensional content. However, as illustrated by the shaded background, the scalar fields (e.g., temperature, composition, and density) depend only on the axial position and are radially independent. Most applications of practical interest involve heat and mass transfer, as well as chemical reactions [35,38,39]. The scalar fields remain radially independent even when homogeneous chemistry is involved. It is the inherent radial independence that enables the desired deposition uniformity in chemical vapor deposition processes.

Fig. 1 shows an inlet manifold and stagnation surface with finite radial dimensions. Of course, any real reactor must have finite dimensions as well as external walls that confine the reactive fluids. However, the present analysis is based on ideal stagnation flow, assuming infinite radial extent of the inlet and the stagnation surface. Fortunately, to a very good approximation, real reactors can be designed and operated in regimes that closely reproduce the ideal flow [40–49].

The following Section presents the governing equations in a general setting and in dimensionless similarity form. As is usually the case, the dimensionless equations depend on characteristic length and velocity scales. The present formulation choses four sets of characteristic scales to represent the semi-infinite and finite-gap configurations, with and without stagnation-surface rotation. The Nusselt and Sherwood numbers follow from the computational solution of the dimensionless differential equations. Correlated results are found to depend on a composite Reynolds number that combines rotating-disk and stagnation-flow behavior.

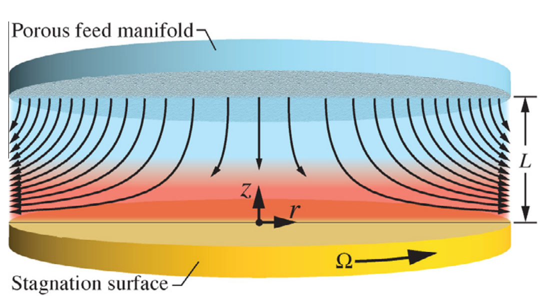

Fig. 1. Illustration of a finite-gap stagnation-flow reactor, where the lower stagnation surface may rotate.

When considering incompressible fluids, the analysis assumes constant properties (conductivity, viscosity, diffusion coefficients, heat capacity). In the case of ideal gases, the Prandtl and Schmidt numbers are presumed to be constants, which is a very good assumption. Because the Prandtl number for gases remains essentially constant there is no specific restriction in the formalism on the temperature dependence of the individual contributing properties. In fact, they may take their usual temperature dependences, with virtually no compromise of the model assumptions.

2. Governing equations

Beginning with the full steady-state axisymmetric Navier–Stokes equations (including thermal and species conservation), Kee, et al. [29] provide detailed derivations of the stagnation-flow equations in boundary-value form. In brief, there are two underlying postulates. First, assume that the streamfunction Ψ can be represented in a separable form as

$$\Psi(z,r) = r^2 U(z), \tag{1}$$

where $U(z)$ is an as-yet unspecified function of z alone. Using the definition of the axisymmetric streamfunction, it follows directly that

$$\frac{\partial \Psi}{\partial r} = \rho u r = 2rU, \quad -\frac{\partial \Psi}{\partial z} = \rho v r = -r^2 \frac{dU}{dz}, \tag{2}$$

where ρ is the mass density and u and v are the axial and radial velocities, respectively. Second, assume that temperature and species composition are functions of z alone (cf., Fig. 1). With these assumptions, the axisymmetric Navier–Stokes equations can be transformed to a system of ordinary differential equations. Then, there must be a set of boundary conditions that do not contradict the assumptions. Fortunately, this is the case. For the purposes of the present paper, homogeneous gas-phase chemistry is neglected. Without repeating the derivations, the system of governing equations can be written as:

Mass continuity:

$$\frac{d(\rho u)}{dz} + 2\rho V = 0, \tag{3}$$

Radial momentum:

$$\rho u \frac{dV}{dz} + \rho\left(V^2 - W^2\right) = -\Lambda_r + \frac{d}{dz}\left(\mu \frac{dV}{dz}\right), \tag{4}$$

Circumferential momentum:

$$\rho u \frac{dW}{dz} + 2\rho V W = \frac{d}{dz}\left(\mu \frac{dW}{dz}\right), \tag{5}$$

Thermal energy:

$$\rho u c_p \frac{dT}{dz} = \frac{d}{dz}\left(\lambda \frac{dT}{dz}\right), \tag{6}$$

Species continuity:

$$\rho u \frac{dY}{dz} = \frac{d}{dz}\left(\rho D \frac{dY}{dz}\right), \tag{7}$$

The independent variable z is the height above the stagnation surface. The dependent variables include the axial velocity u and the scaled radial velocity $V = v/r$, where v is the radial velocity and r is the radial coordinate. The scaled circumferential velocity is $W = w/r$, where w is the circumferential velocity. The temperature is represented as T and Y is the mass fraction of a trace species in a chemically inert carrier. In the case of a gas, the pressure p and density ρ are related via the ideal-gas equation of state.

$$p = \rho R T. \tag{8}$$