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Transient conjugate heat transfer in critical flow nozzles

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ABSTRACT

The body temperature of critical flow nozzle is cooled by expanding cold gas. Subsequently, the thermal boundary layer and throat area are influenced by the conjugate heat transfer at the solid–fluid interface which is called thermal effect. This conjugate heat transfer process in nozzle flow with shock-induced separation were investigated experimentally and numerically, involving three-dimensional wall conduction and fluid convection. Numerical computations solved Reynolds-averaged equations based on SST k–ω model coupling with solid-phase heat conduction equation and were validated by some experiments. Three-dimensional separation criteria and the asymmetry of body temperature were investigated. The maximum asymmetry of body temperature appears at $d = 5.25$ mm and $p_0 = 4.5$ bar. For this asymmetric flow, the experimental isotherm upstream of separation point obtained by twelve temperature points in different sides is accuracy which is enough to study the thermal effect and downstream isotherm with a maximum error of 0.5 °C is mapped merely for reference. At steady-state, minimum temperature point is close to separation point rather than nozzle exit. The body temperature gradually drops with the increase of throat diameter and inlet pressure. The maximum body temperature drop can reach to 15.0 °C. The detailed process of conjugate heat transfer was analyzed by Mach contour in fluid region, and isotherm, heatline in solid region. Finally, the dynamic response characteristics, especially thermal time constant of the body temperature were discussed.

1. Introduction

Critical flow (supersonic) nozzles are mass flow instruments designed for accurate flow measurement, flow controller and flow limiting in many diverse applications by aerospace, automotive, energy and metrology industries [1,2]. Along with the drop of expanding gas temperature, the nozzle body will be cooled by cold gas. Subsequently, thermal boundary layer [3,4] and throat area are influenced by the conjugate heat transfer at the solid–fluid interface, especially for small- or micro- nozzle. This phenomenon is called thermal effect which had been experimentally investigated by Li [5], Wright [6], Thomas [8], Bignell [9], and Ünsal [10].

The actual mass flow-rate of critical flow nozzle Q_m is calculated by [6,7]

$$Q_m = [C_d + (C_T - 1)]\frac{C_*(C_\alpha A_t)p_0}{\sqrt{R_m T_0}} \tag{1}$$

where, C_d and C_* are discharge coefficient and critical flow factor at adiabatic wall. p_0 and T_0 are inlet stagnation pressure and temperature respectively. A_t and R_m represent nozzle area and specific gas constant. C_T is correction factor for the thermal boundary layer when body (wall) temperature $T_w \neq T_0$. C_α is correction factor for the thermal deformation of the throat area. When the body temperature T_w controlled by an electric heater is uniform, C_T is expressed as $1 - K\,Re_t^{-0.5}[\Delta T/T_0]$, where $\Delta T = T_w - T_0$, Re_t is throat Reynolds number and K ranges from 5.05 to 7.05 [6]. Nevertheless, during the normal operation, the body temperature is non-uniform and transient. For the purpose of acquiring accurately the thermal effects on mass flow-rate of the nozzle, the first problem is to figure out the transient conjugate heat transfer in critical flow nozzle.

However, this conjugate heat transfer problem has not yet been thoroughly solved due to the complicated flow pattern with shock-induced separation and various convection heat-transfer coefficient [11]. A large amount of the literatures focus on prediction of the shock structures and separation patterns in supersonic nozzle [12–14]. There are two shock structures, symmetric and asymmetric shocks [15], and two separation patterns, free-shock separation (FSS) and restricted shock separation (RSS) [16], as experimentally and numerically observed by Xiao [15], Hagemann [17], and Ostlund [18] in overexpanded nozzle flow. For predicting the separation location, lots of scholars, such as Summerfield, Kudryavtsev and Romine presented many empirical models for separation criteria [19,20]. Whereas, most studies were performed on conical and truncated ideal nozzles only for FSS [21].

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Nomenclature

A	area, m^2
a	sound speed, $m \cdot s^{-1}$
Bi	Biot number, $= hL/\lambda_f$, –
C_*	critical flow factor, –
C_d	discharge coefficient, –
C_f	skin friction coefficient, –
C_T, C_α	correction factors, –
C1, T1, L1	reflected shock, triple point, slip line
c_p	isobaric heat capacity, $J \cdot kg^{-1} \cdot K^{-1}$
d	nozzle throat diameter, mm
E	total energy, $= c_pT + u^2/2$, $J \cdot kg^{-1}$
F_1, F_2	the blending functions, –
Fo	Fourier number, $= \alpha t/L^2$, –
FSS, RSS	free shock separation, restricted shock separation
G_k, G_ω	generation of k and ω due to mean velocity gradients
H	nozzle height, m
h	convective heat-transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
I	turbulence intensity, %
i	specific enthalpy, $J \cdot kg^{-1}$
K	a constant slope of C_T, –
k	turbulence kinetic energy, $J \cdot kg^{-1}$
L	characteristic length, m
Ma	Mach number, $= u/a$, –
NPR	nozzle pressure ratio, $= p_0/p_a$, –
P_r	Prandtl number, $= \mu c_p/\lambda$, –
p	pressure, Pa
Q_m	actual mass flow-rate, $kg \cdot s^{-1}$
q_s, q_f	wall heat flux, $W \cdot m^{-2}$
R_c	radius of curvature, m
Re	Reynolds number, –
R_m	specific gas constant, $J \cdot kg^{-1} \cdot K^{-1}$
r, φ, X	cylindrical coordinates
S_u, S_E, S_i	Source terms for Eqs. (3), (4) and (13)
St	Stanton number, –
T	temperature, K
T_{aw}	adiabatic wall temperature
t	time, s
u	velocity, $m \cdot s^{-1}$
V	volume, m^3
X, Y, Z	Cartesian coordinates
Yk, Y_ω	dissipations of k and ω due to turbulence

Greek

α	thermal diffusivity, $m^2 \cdot s^{-1}$
β	nozzle diffuser angle, °
Γ	the blending factor, –
γ	isentropic exponent, –
ΔT	body temperature drop, $= T_w - T_0$, K
δ_1	displacement thickness, m
ε	dissipation rate, $m^2 \cdot s^{-3}$
θ	temperature difference, $= T - T_\infty$, K
λ	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
μ	dynamic viscosity, $Pa \cdot s$
ρ	density, $kg \cdot m^{-3}$
σ_κ, σ_ω	turbulent Prandtl numbers for k and ω, –
$\tau_{i,j}$	deviatoric stress tensor, Pa
τ_w	wall shear stress, Pa
τ	time constant, s
ψ	shock angle, deg
Ω	strain rate magnitude, s^{-1}
ω	specific dissipation, s^{-1}

Subscripts

0	stagnation condition
1	main-stream
a	ambient
aw	adiabatic wall
cr	critical condition
d	design
e	nozzle exit
eff	effective
f	fluid
HD	hydraulic diameter
i, ∞	initial and final states
i, j	tensor notation
min, max	minimum, maximum
p	plateau
ref	reference
s	solid, shock
sep	separation
t	nozzle throat/turbulent
w	nozzle wall (body)

Additionally, even though the methodology of conjugate heat transfer is not new, it is only in the last decades that it became more popular due to improvement of computational power and technology [22]. Nowadays, numerical methods have been successfully applied to various fields, such as [23], thermocouple sensor [22], cylindrical tube [24], and supersonic flight [25]. Lin and Kuo [26], Schutte et al. [27], Bilir [28,29], Yang and Tsai [30] made a great deal of numerical researches about transient conjugate heat transfer problems. Besides, there also are several experimental studies on this issue, such as unsteady and conjugate heat transfer in thin liquid-film flows by Mathie [31,32] and Markides et al. [33]. These results could guide our research on critical flow nozzle. Actually, the critical flow nozzle has been investigated by numerical simulation [34].

The present study focused on the transient conjugate heat transfer of supersonic flow with shock-induced separation in critical flow nozzle, using experimental, theoretical, and computational approaches. A three-dimensional fluid solver based on shear-stress transport (SST) k–ω model for supersonic nozzle flow was coupled with a solid-phase heat transfer solver. The simulation was in agreement with experimental data. More details about shock-induced separation criteria and the characteristics of conjugate heat transfer in critical flow nozzle were presented.

2. Problem description

According to regulation of ISO 9300 [35], the geometry of critical flow nozzle (3D) is rotationally symmetric and it has a convergent inlet with radius of curvature twice throat diameter $R_c = 2d$ followed by a conical outlet with constant diffuser angle β. The heat and fluid flows are shown in Fig. 1.

2.1. Shock separation pattern

At moderate nozzle pressure ratio (NPR), a shock occurs inside the nozzle and the downstream flow will separate from the nozzle wall (separation point). Two possible structures, symmetric and asymmetric shocks can be observed in separation flow, as shown in Fig. 2.

Summerfield [36] firstly reported that the flow separation in a planar nozzle was asymmetric at low NPR, but no model was pre-