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Design criteria for improving insulation effectiveness of multilayer walls

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A B S T R A C T

Energy savings in buildings can be achieved to a large amount by optimizing the insulation capabilities of its external walls. Typically such walls are multilayer structures whose insulation properties have a crucial dependence on the type, thickness and ordering of the materials employed in their construction. In this work, the heat transfer matrix approach is used for a quantitative analysis of heat conduction in one-dimensional multilayer structures under steady periodic conditions. In this way, some of the results previously suggested by numerical data are accounted for in a rigorous way. In particular, a fundamental inequality concerning layer order and its effect on the modulus of the temperature decrement factor introduced by a wall is derived. This provides a design criterion for building effective insulation structures. Another factor that can significantly affect the insulating performance of a wall is the number of employed layers, once the total extension of the wall, the total material amounts, and therefore the total thermal resistance of the wall, have been fixed. It is shown that, under specific circumstances, an optimum number of layers can be identified. The influence of layer order and distribution on the time delay that a wall introduces between outer and inner temperatures is also addressed. The presented results are illustrated by means of numerical examples that show how to control to a large extent modulus and phase of the temperature decrement factor of a multilayer wall.

1. Introduction

Energy needs of buildings represent 40% of overall energy consumption in Europe [1]. In particular, heating and cooling demands account for approximately 70% of building energy requirements. The possibility of reducing such needs is therefore of paramount importance, not only with an energy saving objective, but also in the continuous effort to reduce greenhouse gas emissions [2]. In turn, this constitutes a piece of a larger mosaic that requires also to address the sometimes conflicting issue of sustainability [3–5].

Improving the insulation capability of a building structure and in particular of its external walls is one of the most effective means to reduce its energy needs [6–11], an objective that can be pursued both for new and existing buildings [12,13]. While, for a given total thermal resistance, the steady-state behavior of buildings' walls is obviously independent on the relative location, thickness and distribution of the layers constituting the wall, daily and seasonal temperature variations may be instead significantly affected by such geometrical parameters. This issue has been the subject of a large amount of works devoted to investigate how to optimize temperature's damping and delay introduced by a multilayer wall [14–24]. The works cited above fall into two categories: they may study the behavior of the multilayer wall independently of the reference to specific climatic conditions [14,16,17,21,22] or they can include them in the analysis [15,18–20,23,24]. In the first case the boundary condition that may be applied is an adiabatic one, i.e. internal heat flux oscillations are assumed to vanish, which allows decoupling the system from actual external temperatures and solar radiation contributions and focusing on the behavior of the wall on its own. The reference to specific climatic situations and/or wall orientation further specifies the problem, but it is also a fact that prevents the possibility of easily comparing and extending the obtained results and drawing general conclusions on the behavior of the building envelope. For this reason, we select in this work the first approach, which allows deriving general indications and trends that can be, in a further step, applied and refined with the inclusion of the influence of climatic data.

Literature studies usually agree on the fact that the best position of an insulating layer is closer to the heat source (a configuration that has the additional advantage of providing easier prevention of moisture condensation [25]). It turns out also [19,22] that increasing the partitioning of layers, by keeping the same total amount of insulating and conducting material, may improve the wall performance. The problem is generally treated by solving the heat conduction equation, with proper boundary

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Nomenclature

c_p	constant-pressure specific heat
C	heat capacity
D	thermal diffusivity
F_{dec}	temperature decrement factor
G_{F}	temperature gain factor
G_{F}^q	heat flux gain factor
K	thermal wavenumber
L	layer thickness
p	outer fraction of a split layer
q_0	outer heat flux variation
q_L	inner heat flux variation
R	thermal resistance
t	time
T_0	outer temperature variation
T_L	inner temperature variation
Y	thermal admittance

Greek symbols

α	conductor thickness by total thickness
β	damping factor
γ	adimensional parameter
δ	adimensional parameter
θ	ratio of thermal admittances
λ	thermal conductivity
ρ	density
τ	time constant
ω	angular frequency

Subscripts and superscripts

c	conductor
i	insulator
opt	optimal value

conditions, either numerically (implementing finite-difference methods [14,17,19,20,24] or with existing dynamic codes [15,23]), by complex finite Fourier transform [18] or by resorting to electrical analogy [16,21,22]. When employing this latter approach, which makes use of lumped electrical circuit elements to model the thermal behavior of the multilayer structure, one must be aware of the possible inaccuracy arising from the use of a limited number of circuit elements [26,27]. A systematic and rigorous approach to prove some of the conclusions that are common to most of these studies is however missing.

In this work we analyze such issues by applying the heat transfer matrix description of heat conduction in a one-dimensional structure subject to sinusoidal temperature variation [28]. The availability of an exact analytical treatment of this problem will allow deriving interesting results that are easily generalizable and useful for an optimized design of the building envelope. In particular, we will focus on the temperature decrement factor between inner and outer temperature introduced by a wall and we will derive a fundamental inequality for building element order, which explains the conclusions of several literature investigations but was never actually proved up to now. We will also address the possible improvement in terms of insulation obtained with layer partitioning by showing when this is actually advantageous and by highlighting the existence of an optimum number of layers.

The objective of this work is to study how to maximize thermal insulation: although there are cases in which this might not be the most effective solution for reducing energy consumptions [29], it is usually observed that increased insulation is beneficial for reducing overall heating and cooling demands.

While the main application of this study is building envelope design, there are several other fields like, e.g., non-destructive measurements [30,31], aerospace [32] or biomedical applications [33], where the control of heat conduction in layered structures is necessary and the results shown here could prove useful.

2. Steady periodic heat conduction in a multilayer wall

We recall here some basic equations that are used to describe, in a one-dimensional model, the behavior of a multilayer wall subject to harmonic temperature variation. The one-dimensional heat conduction problem in a homogeneous wall of thickness L, thermal conductivity λ and thermal diffusivity D, subject to harmonic temperature variation on its boundary faces can be analytically described [28,34] by linking temperatures and heat fluxes on its faces by a transfer matrix in the following way:

$$\begin{bmatrix} T_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} \cosh(KL) & \sinh(KL)/(\lambda K) \\ \lambda K \sinh(KL) & \cosh(KL) \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix} = \begin{bmatrix} C & S/Y \\ YS & C \end{bmatrix} \begin{bmatrix} T_L \\ q_L \end{bmatrix}, \quad (1)$$

where T_0 and q_0 are the amplitudes of, respectively, temperature and heat flux oscillations at the outer face, and T_L and q_L the corresponding ones at the inner face, $K = (1+j)\beta = (1+j)\sqrt{\omega/(2D)}$ and $\omega = 2\pi f$ is the angular frequency (typically considered to be that of the daily temperature variation: $\omega = 7.27 \cdot 10^{-5}$ rad/s). A shorthand notation to be used in the following has also been introduced, with $Y = \lambda K$ denoting a thermal admittance. It is worth noting that the transfer matrix in Eq. (1) has unity determinant, a property that reduces to three the number of independent matrix elements. This of course is still valid for a multilayer wall where the determinant of the overall matrix is the product of the matrix determinants of the single layers.

Usually the effect that the massive wall has on inner temperature variations is quantified by a temperature decrement factor defined as $F_{\text{dec}} = T_L/T_0$. In the following we will also use its reciprocal value, which we term the gain factor $G_{\text{F}} = T_0/T_L$, as it will be more convenient to employ in the analytical treatment. The decrement and the gain factors are complex quantities that take into account both an amplitude variation and a phase lag (illustrated in the following by a corresponding time delay between outer and inner temperatures). If an adiabatic boundary condition is selected ($q_L = 0$), $G_{\text{F}} = \cosh(KL)$. Similarly, a heat flux gain factor $G_{\text{F}}^q = q_0/q_L$ can be defined, which is simply $G_{\text{F}}^q = \cosh(KL)$ if an isothermal boundary condition is selected at the inner face ($T_L = 0$).

For small values of $|KL|$, a second-order Taylor expansion of the $\cosh(\cdot)$ function gives in the adiabatic case $G_{\text{F}} \simeq 1 + K^2L^2/2 = 1 + j\omega RC/2$, where $R = L/\lambda$ and $C = \rho c_p L$ have been introduced, with ρ the volume density and c_p the specific heat of the homogeneous wall. In this picture, R and C just define a thermal resistance and a heat capacity of the wall but they do not imply any particular lumped-parameter scheme to describe the wall behavior. In the general case (no restriction to adiabatic boundary condition), by approximating both terms in the expression of T_0 through their