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Transient energy growth analysis of a thermoacoustic system with distributed mean heat input

Lei Li [a,b], Dan Zhao [a,*], L.P.H. de Goey [c]

[a] School of Mechanical & Aerospace Engineering, College of Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore
[b] Key Laboratory for Power Machinery and Engineering of Ministry of Education, Shanghai Jiao Tong University, Shanghai 200240, China
[c] Department of Mechanical Engineering, Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, Netherlands

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ABSTRACT

Transient growth of flow disturbances has great potential to trigger unwanted thermoacoustic instability. So far transient growth analysis has tended to focus on thermoacoustic systems with acoustically compact heat sources, even though many systems are associated with distributed mean heat input, such as a premixed flame. In this work, transient growth analysis of both choked and open-ended thermoacoustic systems in the presence of a mean flow and a spatially distributed mean heat input is conducted. Unsteady heat release is modeled within the classical time-lag $\aleph$–τ formulation. Both uniform and triangular distributions for the rate of mean heat input are considered. The generation of entropy disturbances with such distributed heat input is studied first. It is shown that the entropy waves generated by the uniform and triangular distributed heat input are increased first and then decreased with increased frequency. This is different from the conventional concentrated heat input, of which the entropy waves produced is frequency-independent. In addition, the entropy eigenmodes are shown to be non-orthogonal. To quantify transient growth of flow disturbances, two energy measures are defined, calculated and compared. One is associated with the conventional acoustical energy. The other is associated with both acoustic and entropy disturbances. It is shown that the maximum transient growth G_{ac}^{max} of acoustical energy is in the range of 10^2–10^3 in the choked system, while $10^0 \leqslant G_{ac}^{max} \leqslant 10^1$ in the open-ended system. Furthermore, the longer of the uniform distributed heat input, the larger G_{ac}^{max}. However, such finding is not observed for the triangular heat input. Further insights are obtained by examining the contribution of eigenmodes in different frequency ranges. It is found that the lower frequency eigenmodes play a dominant role. Finally, the effect of the interaction index $\aleph$ on transient growth is examined. It is found that the maximum transient growth of acoustical energy G_{ac}^{max} and total energy G_{tot}^{max} are decreased with increased $\aleph$. It is also found that the longer of the uniform distributed heat input, the lower G_{tot}^{max}. These findings are consistent with those obtained in our non-orthogonality and entropy generation analyses.

1. Introduction

Thermoacoustic instability arises from the coupling of acoustics and unsteady heat release [1–4]. It is characterized by large-amplitude pressure oscillations [5–7]. And these self-sustained oscillations occur in many combustion systems [8–12] or in thermoacoustic engines [13–15]. As a conventional tool for studying the stability behavior of a thermoacoustic system, linear stability analysis is generally conducted. In such analysis, eigenfrequencies are calculated. A thermoacoustic system is linearly stable, when all eigenmodes decay exponentially. However it is found that a thermoacoustic system is generally associated with non-orthogonal eigenmodes [16]. And the acoustical energy may undergo transient growth. The non-orthogonality and transient growth of a thermoacoustic system is investigated by different research groups [17,18], and received more attention recently [19–21]. In fact the transient growth plays an important role in triggering the non-linear instability in a 'linearly stable but non-linearly unstable' system [21,22].

Thermoacoustic systems have been shown to be non-normal. And transient growth of acoustic disturbances is caused by unsteady heat release [17,18], non-trivial boundary conditions [23] and the temperature ratio across the heat source [19]. In most of previous works, the flow disturbances in thermoacoustic system are expanded by using Galerkin modes [17–19]. One of the

* Corresponding author.
E-mail address: zhaodan@ntu.edu.sg (D. Zhao).

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drawbacks associated with that method is that the mean flow effect needs to be ignored. However, this is not consistent with practical combustion systems, in which a mean flow is present and entropy disturbances may be generated and propagating.

The entropy disturbances are known as 'indirect combustion noise', which can generate acoustic waves in the accelerating mean flow [24]. The coupling of entropy and acoustic and other flow disturbances has been widely investigated on choked nozzle/combustors [25–28]. Recently researchers have found that the entropy disturbances play an important role in generating thermoacoustic instability [3,29–33]. Nicoud and Poinsot [31] argued that the Rayleigh criterion should be revised to include the entropy terms, when a mean flow is present. And the flow disturbances energy should include the entropy terms [34,35]. The entropy disturbances also have significant effects on the transient growth analysis of a thermoacoustic system. Wieczorek et al. [23] investigated the transient growth of the total energy of both entropy and acoustic perturbations present in the system, based on the Myers' energy norm [35].

In the previous works concerned with the transient growth, the heat sources confined in thermoacoustic systems were always assumed to be concentrated in a plane, i.e., $d/L \to 0$. Here d denotes the length of the heat source region. L denotes the length of the combustor. But in many practical systems the mean heat input is distributed over a significant length [3]. In fact the distribution of the heat input/source has been shown to affect the prediction of thermoacoustic instabilities. Kim et al., [36] showed that the predicted eigenfrequency by using a distributed FTF (flame transfer function) agrees with experimental measurements better than that using a compact FTF. Dowling [3] has shown that the eigenfrequencies of a thermoacoustic system were significantly influenced, when d is comparable with the length scale of the entropy wave, i.e., $d \sim \frac{\bar{u}}{\omega} \ll \frac{c}{\omega}$ (acoustic wavelength), Here ω and $\bar{u}$ denote the frequency and the mean velocity respectively. It was found that the fundamental eigenfrequency associated with a distributed heat source with a length $d/L = 0.25$ is about 100% larger than that with a concentrated heat input. In fact compared with $d/L \to 0$, across the region of the heat source the quantities of acoustic and entropy waves are strongly influenced by the distribution of heat input if $\omega d/\bar{u} \ll 1$ is not satisfied. The transient energy growth of a system is largely dependent on the energy of flow disturbances [3]. Hence it is interesting to know whether a distributed heat input may have significant effects on the transient growth analysis of a thermoacoustic system in the presence of a mean flow. This partially motivates the present work.

To characterize transient growth, an energy measure is needed. There are two general energy measures defined in a thermoacoustic system in the presence of a mean flow [17,18,23,34,35]. One energy measure is concerned with conventional acoustical energy. The other measure involves with the total energy of both entropy and acoustic disturbances. The transient growth of the total energy was studied in Ref. [23]. It has been reported that for some cases non-linear thermoacoustic instability may be triggered if the intensity of the acoustic disturbance is large enough [22]. This indicates that transient growth of acoustical energy is also important.

In this work, transient growth analysis of a choked and an open-ended thermoacoustic system in the presence of a mean flow and spatially distributed heat input is conducted. The configuration and flow conditions are described in Section 2. Modal analysis of the thermoacoustic system is then conducted in Section 3. In our analysis, the mean heat input region is divided into several subdomains. Validation of the thermoacoustic model and the discretization method is then performed. In Section 4, the generation of entropy disturbance is studied. And the non-orthogonality analysis of the entropy eigenmodes is conducted in Section 5. Then transient energy growth analysis of the thermoacoustic systems with uniform and triangular distributed heat input is performed. This is described in Section 6. The two different energy measures are compared.

2. Description of thermoacoustic systems with distributed mean heat input

A one-dimensional thermoacoustic system in the presence of a mean flow is considered in the present work as shown in Fig. 1. The mean heat input is distributed over a region d in between $x = L_f - d/2$ and $x = L_f + d/2$. The center of the heat source region is located at $x = L_f$. The outlet boundary is either choked [37] (see Fig. 1(a)) or open-ended (Fig. 1(b)). The main length of the duct is L. It is assumed that the flow is isentropic and uniform in the pre- and post-heating regions. However, flow parameters are varied in the heat input region. Subscripts 1 and 2 denote the upstream and downstream regions with respect to the heat input, respectively. ρ, u, p, T, M, c denote the density, velocity, pressure, temperature, Mach number and sound speed in the system, respectively.

3. Modal analysis of thermoacoustic systems with distributed mean heat input

3.1. Coupling between entropy and density fluctuations

The flow parameters consist of a mean part denoted by an overbar and a fluctuating one denoted by a prime, e.g., $p = \bar{p} + p'$. The flow disturbances can then be expanded by using traveling waves as [38]:

$$p' = (P^+ e^{i\alpha^+ x} + P^- e^{i\alpha^- x}) e^{i\omega t}, \tag{1a}$$

$$u' = -\frac{1}{\bar{\rho}\bar{c}} (P^+ e^{i\alpha^+ x} - P^- e^{i\alpha^- x}) e^{i\omega t}, \tag{1b}$$

$$\rho'_a = \frac{1}{\bar{c}^2} (P^+ e^{i\alpha^+ x} + P^- e^{i\alpha^- x}) e^{i\omega t}, \tag{1c}$$

where p', u', and ρ' denote the fluctuations of pressure, velocity and gas density. The subscript a denotes acoustic quantities. P denotes the magnitude of the traveling waves. Overhat + and − denote the waves propagating upstream and downstream, α is the wave number and it is given as $\alpha^+ = \frac{\omega}{\bar{c} - \bar{u}}$ and $\alpha^- = -\frac{\omega}{\bar{c} + \bar{u}}$, where $\omega = \omega_r + i\omega_i$ denotes the complex eigenfrequency. The real part ω_r represents

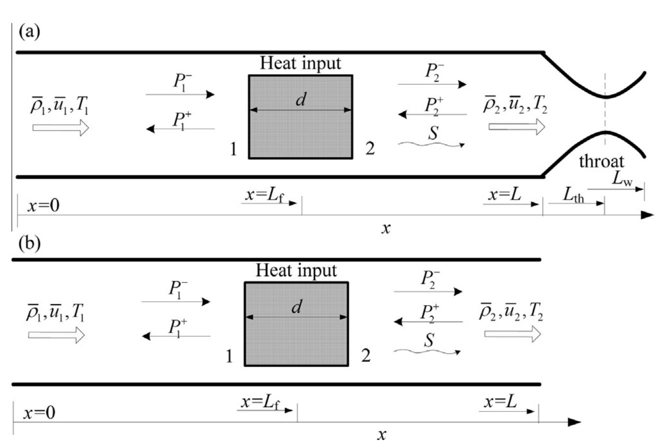

Fig. 1. Schematic of the one-dimensional thermoacoustic system with distributed mean heat input, (a) choked outlet; (b) open-ended outlet. The geometric dimensions of the choked nozzle are given as $L = 1.0$ and $L_{th} = 1.08$.