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Theory for the scaling of metal temperatures in cooled compressible flows

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ABSTRACT

The scaling of metal temperature in conjugate heat transfer problems in compressible environments is argued from first principles in this paper, and a new invariant temperature parameter is proposed. The objective is to provide a framework for comparing metal temperatures between different temperature boundary conditions while appropriately dealing with compressibility effects. The theoretical analysis is based on the principle of superposition, applicable due to the linearity in temperature of the governing differential equation: the time-averaged general viscous energy equation. It is demonstrated that if all non-dimensional parameters that govern the flow field are equal for different temperature boundary conditions, then the overall cooling effectiveness, ϕ, is also equal if defined as

$$\phi = \frac{T_{w1} - T_{\Re}}{T_{02} - T_{\Re}}, \tag{1}$$

where T_{w1} is the external wall temperature, T_{02} the coolant inlet total temperature (before interaction with any part of the metal surfaces), and $T_{\Re}$ a local *'recovery and redistribution temperature'*. As an overall cooling effectiveness thus defined is independent of mainstream and coolant temperatures, it is shown that it is the correct parameter to scale experimental test rig measurements to full engine conditions. A numerical model of a reverse pass cooling system, internally and externally cooled and representative of typical turbine geometries, has been developed to support the theoretical analysis, aid interpretation of the parameters ϕ and $T_{\Re}$, and provide a demonstration of the scaling theory for a relatively simple flow field. The arguments are presented for a system in which the physics can be prescribed, and therefore fully controlled, to emphasise the physical reasoning behind the newly proposed parameters.

1. Introduction

Invariant descriptors of convective heat transfer are required to characterise thermal loads in components which operate in environments of significant forced convection. Ideally, adequate descriptors need to deal with non-uniform thermal boundary conditions, including those induced by conjugate heat transfer effects [1]. This paper focuses on the high pressure turbine nozzle guide vane to make the discussion more specific, one of such components where forced convection (in the form of internal and external cooling) is employed to mitigate the severe external thermal load, also predominantly convective. The theory presented has wider applications, however. It will be demonstrated that the overall cooling effectiveness, defined with respect to a newly proposed recovery and redistribution temperature, is the relevant invariant descriptor in conjugate heat transfer problems in cooled compressible flows.

The heat transfer problem in turbine nozzle guide vanes is governed by the general viscous energy equation. As set out in many references [2,3], in the case of one gas species and no sources of thermal energy, it may be expressed in vector notation as

$$\rho \frac{Dh}{Dt} - \nabla \cdot k \nabla T - \mu \Phi - \frac{Dp}{Dt} = 0, \tag{2}$$

where D/Dt refers to the substantial derivative, given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla. \tag{3}$$

When applied to the flow of heat over turbine vanes, a number of justifiable first-order approximations may be employed, which allow for a discussion of the solution to the differential equation from its mathematical properties. First, the viscous dissipation

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Nomenclature

A, B	external heat transfer coefficients, W/m^2 K
A^*, B^*	external heat transfer coefficients, W/m^2 K
C, C', C^*	internal heat transfer coefficients, W/m^2 K
C_P	specific heat capacity at constant pressure, J/kg K
h	heat transfer coefficient (general), W/m^2 K; Specific enthalpy, J/kg
k	thermal conductivity, W/m K
L	characteristic length, m
$\dot{m}_c$	coolant mass flow rate per unit width, kg/s m
M	Mach number
n	surface normal coordinate, m
Nu	Nusselt number, $Nu = hL/k$
p	static pressure, N/m^2
p_0	total pressure, N/m^2
Pr	Prandtl number, $Pr = \mu C_P/k$
q	convective heat flux per unit area, W/m^2
r	conventional recovery factor, $r = f(Pr)$
$\Re$	recovery and redistribution parameter, $\Re = T_{\Re}/T_{01}$
s	surface tangential coordinate, m
t	thickness, m
t	time, s
T	static temperature, K
T_0	total temperature, K
T_r	conventional recovery temperature, K
$T_{\Re}, T_R$	recovery and redistribution temperature, K
$\vec{u}$	velocity vector, m/s
Greek letters	
γ	isentropic exponent of the working fluid
η	film cooling effectiveness
η_f	coolant-to-mainstream mixing fraction
μ	dynamic viscosity, kg/sm
ρ	density, kg/m^3
ϕ	overall cooling effectiveness
Φ	dissipation function, m^{-2}
Subscripts	
∞	incompressible mainstream flow
0	referred to a case without cooling
1	external flow (mainstream)
2	internal flow (coolant)
aw	adiabatic wall conditions
c	referred to the coolant stream
C	solution to the homogeneous differential equation for non-zero coolant temperature
e	turbine vane flow exit conditions
ext	external side of the turbine vane
f	referred to a case with cooling
i	referred to incompressible conditions
in	cooling passage inlet
int	internal side of the turbine vane
local	local variable along the cooling passage
m	mixed-out variable
out	coolant ejection point
P	particular integral
u,U	solutions to the homogeneous differential equation for unit temperature differences
w	referred to the metal wall
W	solution to the homogeneous differential equation for non-zero wall temperature
*w*1	external wall surface
*w*2	internal wall surface

function is ostensibly not a function of temperature, being mostly dependent on the velocity field [3]. In addition, it may be considered that the fluid properties are not a function of temperature, but only of position. This assumption leads directly to the use of Newton's law of cooling and the definition of convective heat transfer coefficients [4,1], and it has been extensively employed in heat transfer research [5] (a correction would be required, however, where this approximation led to significant errors). If, finally, Eq. (2) is time-averaged, the following results:

$$\rho\overline{\vec{u}\cdot\nabla(C_P T)} - \overline{\nabla\cdot k\nabla T} - \overline{\mu\Phi} - \overline{\vec{u}\cdot\nabla p} = 0, \quad (4)$$

which is linear in temperature and can therefore be solved by application of the principle of superposition [6]: the solution of the general equation is the sum of the solution of the homogeneous equation, plus a particular integral of the complete equation.

In incompressible flows of gases, where kinetic energy effects and viscous energy dissipation may be neglected, the time-averaged energy equation is not only linear, but also homogeneous,

$$\rho\overline{\vec{u}\cdot\nabla(C_P T)} - \overline{\nabla\cdot k\nabla T} = 0, \quad (5)$$

and the velocity and temperature fields are decoupled. As a result, the heat flux from a gas stream into the surface of a body immersed in it can be expressed, per unit area, by Newton's law of cooling as $q = h(T_\infty - T_w)$: the product between a heat transfer coefficient solely dependent on the flow field and the local temperature difference between gas and wall.

At high Mach numbers, however, the kinetic energy of gases is significant, the velocity and temperature fields are coupled, and the time-averaged general viscous energy equation is non-homogeneous – Eq. (4). This problem has been traditionally addressed by the use of a recovery temperature, T_r, as the driving temperature for the convective heat flux per unit area, which, in turn, has been expressed as $q = h(T_r - T_w)$. This addresses compressibility effects and viscous dissipation by employing the recovery temperature as the thermal boundary condition (and implies the total temperature is not pertinent in this case). The recovery temperature is defined here as the surface temperature for which there would be no convective heat transfer at the wall. To first order, compressible and incompressible heat transfer coefficients may be assumed to be the same within the subsonic and transonic flow regimes.

2. The influence of cooling

The presence of cooling flows does not alter the linearity of the governing equation in temperature, so superposition arguments may still be employed. Coolant enters into the problem through internal cooling (both convective and impingement) and external film cooling (by being ejected through numerous holes in the turbine vane walls). Generally, however, research into internal and external cooling has been conducted independently, and superposition has been predominantly employed in film cooling investigations [7,8].

The main difference between the case with no cooling and the case with film cooling only (but not internal cooling) arises from the fact that the additional temperature in the problem (that of