International Journal of Heat and Mass Transfer 103 (2016) 582–593

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An approximate analytical method for nonlinear transient heat transfer through a metallic thermal protection system

Liangxian Gu, Yifan Wang *, Shengbo Shi, Cunxi Dai

National Key Laboratory of Aerospace Flight Dynamics, Northwestern Polytechnical University, Xi'an, Shanxi 710072, China

ARTICLE INFO

Article history:
Received 10 May 2016
Received in revised form 25 June 2016
Accepted 21 July 2016

Keywords:
Metallic thermal protection system
Transient heat transfer
Approximate analytical method
Radiation boundary condition

ABSTRACT

Metallic thermal protection system (MTPS) is being investigated more popular in present. In preliminary design, the MTPS can be simplified as a multilayer plate, and a one-dimensional transient heat transfer analysis is commonly used to size the thickness of MTPS. A novel approximate analytical method based on separation of variables and orthogonal expansion technique is presented for temperature prediction through MTPS subjected to convection and radiation boundary conditions, which considers the effects of temperature-dependent thermal material properties. The proposed method applies a set of linear functions with space-variable to describe the distribution of thermal diffusivity through the thickness direction, and it adopts a numerical solution dividing the whole process of heat transfer into a chosen number of time steps to solve the problem of boundary conditions variation with time. An advanced linearized approximation for radiation boundary condition is developed for modified use with the analytical model. Implementation of the proposed method is demonstrated by applying it to determine the problem of nonlinear transient heat transfer through a MTPS made of Titanium 6-4 and Saffil, and a good agreement is presented by comparing with the finite element method (FEM) results.

1. Introduction

The technology development conducts the future launch vehicle to be durable, operable and cost effective. This means that the thermal protection structure not only be able to withstand serious temperature induced by aero-heating during launch and re-entry process, but also should be required to be lightweight and easily maintained. Because the traditional thermal protection system cannot satisfy these requirements, MTPS have been developed to meet these goals [1].

In order to reduce the weight of MTPS, an accurate transient heat transfer analysis is essential to be applied, as it can avoid the conservative design by comparison with a steady state analysis [2]. In addition, a high temperature field induced by serious boundary conditions through the thickness of MTPS is given. Therefore, the effects of temperature-dependent thermal material properties cannot be ignored for the transient heat transfer calculation. The aim of the paper is to find a fast and accurate method to solve the nonlinear transient heat transfer for MTPS.

In general, a MTPS can be simplified as a multi-layered plate, and each layer is assumed as homogeneous in preliminary design [3], therefore, the one-dimensional solution of transient heat transfer analysis for multi-layered system is used to predict the temperature distribution of MTPS. The one-dimensional transient heat transfer analysis can be solved by analytical methods or numerical methods. Analytical methods can provide a better insight into the physical significance of different parameters affecting the problem that are difficult to be obtained from numerical solutions, hence it is always useful in engineering analysis [4]. Otherwise, analytical methods solving the one-dimensional heat transfer through multi-layered plate gives further economy in comparison with numerical methods [5].

Six analytical methods of performing one-dimensional transient heat transfer problem has been presented by de Monte [6]. The methods used by many researchers for analytical transient heat transfer can be usually described as following four categories: the Green function approach [7], the Laplace transform method [8], the separating the variables technique [9] and the finite integral transform technique [10]. The technique of separating the variables is popular for predicting the temperature distribution through multi-layered plate [11]. Using this technique, de Monte [6] developed a solution of transient heat transfer in one-dimensional composite slab by using a 'natural' dimensionless analytic approach, and he also provided an analytic approach for the unsteady heat transfer analysis [12,13]. Computing the eigenvalues

* Corresponding author.
E-mail address: 2293352167@qq.com (Y. Wang).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.07.075
0017-9310/

Nomenclature

F	initial temperature distribution
g	linear function describing thermal diffusivity distribution
h	convection heat transfer coefficient
J_0	zero-order Bessel function of first kind
J_1	first-order Bessel function of first kind
k	thermal conductivity
R	thermal contact resistance
sign	signum function
T	temperature in MTPS
T_∞	ambient temperature
T_w	temperature at external surfaces
t	heating time
X	space-variable function
x	space coordinate
Y_0	zero-order Bessel function of second kind
Y_1	first-order Bessel function of second kind

Greek symbols

α	thermal diffusivity
β	eigenvalue
σ	Stephan–Boltzmann constant $(5.669 \times 10^{-8}\ \text{W/m}^2\ ^\circ\text{C})$
ε	surface emissivity
Γ	time-variable function

Acronyms

MTPS	metallic thermal protection system
FEM	finite element method

to the heat transfer problem with taking into account the contribution of contact resistance was treated by Haji-Sheikh [14]. Based on the work by Sun [15], a full series solution was developed for impulsive heating by employing a 'natural' orthogonal relationship between the eigenfunctions. These analytical solutions for multilayered plate cannot consider the radiation boundary condition, which is known to play a significant role in many heat transfer problems. For instance in Miller's work [16], a linearization of the radiation term is performed in a novel analytic approach for heat transfer through multi-layered plate subjected to convection and radiation boundary conditions, but the linearized approach is limited by the ratio of surface temperature to ambient temperature. Moreover, in view of above-mentioned methods, an assumption is established in study of transient heat transfer that considers the thermal material properties are uniform along thickness of each layer, therefore, it is feasible only in the case of thin layer. However, the distribution of thermal material properties through thickness direction is nonlinear, because of the insulation core of MTPS is regarded as a thick layer and the thermal material properties are temperature-dependent.

Despite a number of analytic solutions have been presented for transient heat transfer problems, only a few investigations can be found for transient heat transfer in a multilayer plate with taking into account the effects of temperature-dependent thermal material properties. An analytic study of the two generalized forms of the nonlinear heat conduction equation was investigated in Kabir's study [17]. Ferraiuolo [18] created a novel hybrid analytical/numerical method to solve transient conduction problem for composite slabs subjected to heat flux and radiation boundary condition, combining Green's function approach with integral method, and the method had the capability of taking into account the effects of temperature-dependent thermal material properties, but the thermal diffusivity was considered as uniform over the phenomenological distance in each time step. Mustafa [19] developed a new method for generating approximate analytic solution of nonlinear transient heat transfer problem based on a combination of Lie symmetry method, homotopy perturbation method, finite element method, and simulation based error reduction techniques, whereas this method was unable to predict the temperature distribution in multilayer plate and consider the radiation boundary condition.

The present paper suggests an approximate analytic method to deal with the nonlinear transient heat transfer for MTPS subjected to convection and radiation boundary conditions, combining the method of separation of variables with the orthogonal expansion technique. In order to take into account the effects of temperature-dependent thermal material properties, an approximate solution is established by using a set of linear functions with space-variable to describe the distribution of thermal diffusivity through the thickness direction. Moreover, a numerical procedure is adopted to solve the problem of boundary conditions variation with time. Based on work by Miller and Weaver [16], an advanced linearized approximation for radiation boundary condition is developed for modified use with the analytical model for convection. At the end of the paper, the accuracy of the present approach is verified by comparison with finite element method (FEM).

2. Linearized approach

The transient heat transfer analysis for MTPS as shown in Fig. 1 is discussed in the paper. In general, the MTPS can be simplified as a sandwich plate, and the external surfaces of MTPS are subjected to convection and radiation boundary conditions. The equivalent sandwich plate is composed of two outer layers made by metallic material and an insulation core filled with thermal protection material. The thickness of insulation core needs to be thick enough to maintain the temperature of underlying structures within acceptable limits, and the outer layers are thin in comparison with the insulation core. According to the characteristic of MTPS, some assumptions are established as

1. The thickness of each layer is sufficiently small in comparison with the length and width of MTPS, and the material of each layer is assumed to be homogeneous;

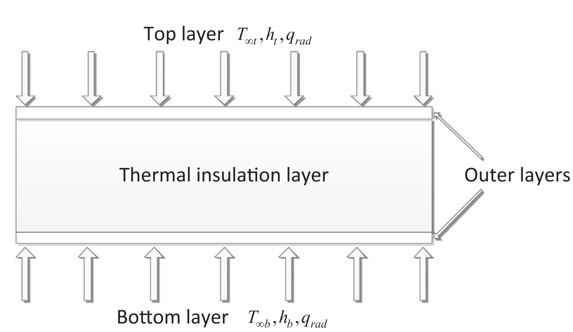

Fig. 1. Schematic representation for transient heat transfer analysis of MTPS.