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Fractional diffusion modeling of heat transfer in porous and fractured media

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ABSTRACT

Fracture–matrix interactions strongly affect anomalous heat transfer in geological sites. This study investigates effects of the interactions between fractures and rock matrix by using the method of *multiple interacting continua* (MINC). The MINC generates different temperature histories for varied fracture spacings. Two analytical solutions of each porous model and fracture model are used to fit the numerical results for temperature histories due to cold-water injection. The porous model is good agreement with the result for small fracture spacing, while a solution of the fracture model fits the result for large fracture spacing. The MINC yields intermediate behaviors in between a porous medium and a single fracture. A fractional heat transfer equation (fHTE) has been developed to describe anomalous thermal diffusion in a fractured reservoir. The fHTE accounts for heat flux from fracture into matrix by using a temporal fractional derivative. The fHTE can capture numerical results for temperature histories with different fracture spacings. The fracture spacing has correlations to the fHTE best-fit parameters (i.e., the orders of fractional derivatives and the retardation parameters). The fHTE with varying time fractional derivatives can cover descriptions of subdiffusion, Fickian diffusion, and superdiffusion.

1. Introduction

Natural fracture networks significantly control hydrodynamic and thermodynamic behaviors in geological fields (e.g. geothermal energy, deep geologic storage of nuclear waste, induced saltwater intrusion, hydraulic fracturing, and carbon dioxide sequestration). Fast flow paths through fractures may lead to rapid migration, while interactions of flow with the rock matrix (i.e., advective imbibition, diffusion, or adsorption) affect the retardation of fluid and heat.

The fracture–matrix interactions are treated with dual-continuum approaches, which include the classical double-porosity model [1,42], the dual-permeability concept [9], and the more rigorous dual-continuum generalization of the method of *multiple interacting continua* (MINC) [19,25]. In the double-porosity concept, a network of interconnected fractures forms the flow paths, and the embedded rock matrix is the subdomains exchanging mass and heat between the flow domain and the stagnant domains. This concept assumes that approximate thermodynamic equilibrium locally exists between fracture and matrix, that is, fracture–matrix exchanges occur instantaneously [8]. In contrast, the MINC method is able to describe gradients of pressures, temperatures, or concentrations inside the matrix by subdividing individual matrix blocks. Although improved capacities of computer simulations allow us to use very large amount of grids and huge computer storage spaces, the main difficulty is to determine numerous site-specific input parameters. Unfortunately, most measurement data are obtained from limited samples and do not accurately describe a fractured medium. Inverse problem analyses (e.g., iTOUGH2 [13,12] and stochastic approaches [41,26]) have been improved but tend to be computationally intensive.

For idealized and simplified systems and conditions, it is possible to solve mathematical models by analytical techniques, which would be attractive to characterize reservoir properties during early phases of developments. Classical modeling for heat transfer in a single fracture treats heat exchange between a fracture and the

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Nomenclature

a	arbitrary constant
a'	thermal diffusivity
b	fracture aperture
C_p	heat capacity
d	arbitrary constant
J	heat flux
K	permeability
k	coefficient including physical properties and the structure of porous rocks
t	time
T	temperature
u	fluid velocity in the fracture
x	horizontal coordinate
X	non-dimensional distance
z	vertical coordinate
β	order of temporal fractional derivative
γ	order of temporal fractional derivative
κ	coefficient depending on the shape of porous blocks
λ	thermal conductivity
λ'	effective thermal conductivity
Φ	memory function
ρ	density
θ	index of permeability reduction

Subscript

0	initial
1	reservoir
c	constant
f	fluid
in	injection
m	matrix
r	rock
s	surrounding rock
w	water

embedded rock matrix. The purpose of this study is to develop a heat transfer equation based on the fractional calculus. Recently, fractional calculus has been applied to modeling methodologies and have attracted interest in several fields, for instance, fluid mechanics [20], rheology [4] bioengineering [24], and hydrological modeling [27,44,2]. Temporal fractional derivatives can be used to describe diffusion into matrix and/or into surrounding rocks where fractures show self-organized fractal distributions [15,16,32,33]. The advantage of using fractional calculus is its ability to characterize phenomena in heterogeneous media with few parameters. The drawback is that the physical meaning of the constitutive parameters is still unknown.

Fractional differential equations for heat transfer has been studied [28,23,10,17]. Little is known about the relationship between the order of time fractional derivatives and geological structures. First, we show conventional modeling approaches for heat transfer and compare with the fractional heat transfer model. Numerical simulation results are obtained from the MINC method to reveal insights into the physical meaning of fractional derivatives in heat transfer.

2. Methodology development

2.1. Conventional mathematical heat transfer models

Bodvarsson [5] derived the basic equation for subsurface temperature fields in a homogeneous porous medium with intergranular flow. The governing equation can be written as:

$$\frac{\partial T_1}{\partial t} = -\frac{\phi_w \rho_w C_{pw}}{\overline{\rho C_p}} u \frac{\partial T_1}{\partial x}, \tag{1}$$

where $\overline{\rho C_p} = \phi_w \rho_w C_{pw} + (1-\phi_w)\rho_r C_{pr}$. T_1 is temperature of the flow domain, t is time, and x is distance. ρ_w and ρ_r are the density of water and rock, respectively. C_{pw} and C_{pr} are the heat capacities of water and rock. ϕ_w is the porosity, and u is the fluid velocity. This equation assumes that uncompressed fluid flows in a homogeneous porous medium. The rock grains are so small that there is a perfect temperature contact between the fluid and the rock grains. Because convection is dominant in most geothermal hydrothermal systems [43], thermal conduction was neglected. This model will be referred to as the *porous model* in this paper. The term on the left hand side describes heat accumulation in the porous medium. The term on the right hand side represents convection.

Lauwerier [21] developed an analytical solution for heat transfer with heat loss into confining beds according to the Fourier law. Heat exchange between a single flowing region (fracture) and stagnant regions is considered. Bodvarsson and Tsang [6] presented a differential equation for a single fracture surrounded by confining rock masses as follows:

$$\begin{aligned} \frac{\partial T_1}{\partial t} &= -\frac{\phi_w \rho_w C_{pw}}{\overline{\rho C_p}} u \frac{\partial T_1}{\partial x} + \frac{\lambda_r}{b\overline{\rho C_p}} \left.\frac{\partial T_s}{\partial z}\right|_{z=0}, \\ \rho_r C_{pr} \frac{\partial T_s}{\partial t} &= \lambda_r \frac{\partial^2 T_s}{\partial z^2}; \end{aligned} \tag{2}$$

where T_1 is the temperature of the fluid in the fracture and T_s is the temperature of the surrounding rock masses. λ_r is the thermal conductivity of the rock and b is the fracture aperture. z is the distance from the fracture, which is perpendicular to the x-axis. The term on the left hand side accounts for heat accumulation in the fracture. The first and second terms on the right hand represent convection in the fracture and heat loss into the confining rocks, respectively. Thermal equilibrium is assumed to take place instantaneously between water and rocks, so that anywhere in the fracture rocks have the same temperature as the surrounding fluid. The heat flux into the surrounding rocks is given by

$$J_s = -\lambda_r \left.\frac{\partial T_s}{\partial z}\right|_{z=0}. \tag{3}$$

This model can express thermal diffusion from a single fracture into the surrounding rocks following the Fourier law. We call this mathematical model the *single-fracture model* in this paper.

2.2. The time fractional diffusion model

Fractional diffusion equation has been used to describe anomalous diffusion processes, which do not follow the Fick's law and are called *non-Fickian* solute transport [27,44]. Fick's law of solute diffusion and Fouriers law of heat conduction both are empirical laws. Fick's law describes that mass flux is proportional to the concentration gradients, while Fouriers law describes that heat flux is proportional to temperature gradients, respectively. A fractional advection–dispersion equation (fADE) can model mass transport