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Lattice Boltzmann Method for simulation of mixed convection of a Bingham fluid in a lid-driven cavity

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ABSTRACT

In this paper, a two-dimensional simulation of steady mixed convection in a square enclosure with differentially heated sidewalls has been performed when the enclosure is filled with a Bingham fluid. The problem has been solved by the Bingham model without any regularisations and also by applying the regularised Papanatasiou model. An innovative approach based on a modification of the Lattice Boltzmann Method (LBM) has been applied to solve the problem. Yield stress effects on heat and momentum transport using the Papanatasiou model are investigated for certain pertinent parameters of the Reynolds number (Re = 100, 500, and 1000), the Prandtl number (Pr = 0.1, 1, and 10) and the Bingham number (Bn = 0, 1, 5 and 10), when the Grashof number is fixed at Gr = 10,000. Results show that a rise in the Reynolds number augments the heat transfer and changes the extent of the unyielded section. Furthermore, for fixed Reynolds and Prandtl numbers, an increase in the Bingham number decreases the heat transfer while enlarging the unyielded section. Although an increase in the Prandtl number enhances heat transfer, it does not affect the proportions of the unyielded/yielded regions in the cavity. Finally, the results of the Bingham and Papanatasiou models are compared and it is found that there is a visible difference between the two models especially in the yielded/unyielded sections.

1. Introduction

Convection involving both free and forced convection is generally referred to as mixed convection, which occurs when buoyancy effects are superposed on a forced flow. Mixed convection in fluid-filled square cavities plays an important role in the area of heat and mass transfer and has been given a considerable attention over the past several years due to the wide variety of applications in science and engineering [1–7]. For example, the flow is present in materials processing, flow and heat transfer in solar ponds, dynamics of lakes, reservoirs and cooling ponds, crystal growing, float glass production, metal casting, food processing, galvanizing, and metal coating. However, most of the research has been limited to incompressible Newtonian fluids, although in a few cases, non-Newtonian fluids have also been considered. Viscoplastic fluids form a special sub-class of non-Newtonian fluids in which the flow field is divided into two regions: the first is an unyielded zone where the fluid is at rest or undergoes a rigid motion, and the second where the fluid flows like a viscous liquid. In the unyielded zone, the second invariant of the extra stress is less than or equal to the yield stress and in the yielded region, this invariant exceeds the yield stress. Thus, the location and shape of the yield surface(s), i.e. the interface between these two sets, is also a part of the solution of flow problems of such fluids. This is a challenging problem and research has been divided into using the Bingham model without any regularisations, or the modification due to Papanastasiou [8], or the bi-viscosity model due to O'Donovan and Tanner [9].

Here, we solve the flow of a Bingham fluid in a lid driven square cavity with differentially heated sidewalls using a new numerical approach, based on the Lattice Boltzmann Method (LBM). This technique is applied to the Bingham model and the Papnasatasiou model so that a comparison between their predictions can be made. As far as the LBM is concerned, it has been demonstrated to be a very effective mesoscopic numerical method to model a broad variety of complex fluid flow phenomena. It has developed into an alternative powerful numerical solver for the Navier–Stokes (N–S) equations applicable to incompressible Newtonian fluids. In comparison with traditional methods in the field, the LBM algorithms are much easier to implement, especially in complex geometries and multi-component flows. This is because the main equation of the LBM is hyperbolic and can be solved locally, explicitly, and efficiently on parallel computers. However, it has had to overcome three main drawbacks in passing from the compressible to incompressible models. The first one arises

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from the existence of a pressure density relation, which is incompatible with flows of incompressible fluids. Several investigators attempted to remove this connection and their procedures introduced other limitations. For example, in the LBM due to [10], although the pressure is an independent variable, the N–S equations for incompressible fluids can only be recovered from the compressible fluid model for very small Mach numbers, and the viscous dissipation effect in the energy equation has to be neglected due to the low Mach number. The second issue is the specific relation between the relaxation time and the viscosity. The third is concerned with solving rheological problems with LBM for non-Newtonian fluids, especially in the presence of the energy equation. Finally, the application of the boundary conditions in LBM is immensely complicated which exhibits the need for more flexible LBM algorithms for boundary conditions.

In this connection, Fu and So [11] proposed a new equation for the equilibrium distribution function, modifying the LB model. Here, this equilibrium distribution function is altered in different directions and nodes while the relaxation time is fixed. In this scheme, the problematic pressure density relation has been eliminated, for the density has a fixed value. The constancy of the relaxation time and the independency of viscosity from the relaxation time lead to the creation of an appropriate method for a wide range of fluids, including non-Newtonian behaviour. However, the Finite Difference Lattice Boltzmann Method (FDLBM) of Fu and So [11] was restricted to fluid flows in the absence of the energy equation. Subsequently, Fu et al. [12] developed a new method for the energy equation and named it the Thermal Finite Difference Discrete Flux Method (TFDDFM) and applied it to solve natural convection flows in two and three dimensional cavities. In addition, in both FDLBM and TFDDFM, boundary conditions can be imposed on the problems in a manner similar to macroscopic methods that enable the application of more flexible boundary conditions, such as a slip condition. In addition, Huilgol and Kefayati [13] have recently derived the continuum mechanics equations of the momentum and energy equations from the FDLBM and the TFDDFM, respectively.

Turning to the study of the flows of viscoplastic fluids, two numerical methods have been found to be useful when the Bingham model is considered. The first is the minimisation of a suitably chosen augmented Lagrangian functional; the second is based on solving a variational inequality; for a summary, see Huilgol [14]. Turning to lid driven cavity flows, Sanchez [15] applied a first-order operator splitting method to solve the corresponding variational inequality for the flow of a Bingham fluid. Subsequently, Dean and Glowinski [16] discussed a numerical simulation of unsteady flows of a Bingham fluid based on time discretisation and illustrated it by applying it to the flow in a lid-driven cavity. Recently, Huilgol and Kefayati [17] employed the operator splitting method to simulate the natural convection in a cavity filled with a Bingham fluid. The numerical method used by them depends on the use of the constitutive equation for the Bingham fluid through the viscoplasticity constraint tensor. This is described fully in Section 2 below.

In addition, in Section 2, the regularised Papanastasiou model is also described for its use subsequently. In the Papanastasiou regularisation model, the fluid is described through a constitutive equation which applies throughout the fluid with a parameter (m) dependent non-Newtonian viscosity. In this model, increasing the parameter causes the fluid to behave more like a Bingham fluid; for instance, Dimakopoulos et al. [18] showed that m may be as high as 10^6. However, most of the simulations use $m = 400$ or 1000, which is also the case here. In connection with the lid driven flow in a cavity, research has been published by Mitsoulis and Zisis [19] and Neofytou [20] which exhibit the streamlines and velocities in the middle of the cavity. More recently, Syrakos [21] has investigated the capabilities and limitations of the popular finite volume/SIMPLE method coupled with the Papanastasiou regularisation for the simulation of the flow in a lid driven cavity. Using the bi-viscosity model, Turan et al. [22] have studied the natural convection problem. However, the flow of a Bingham fluid in a lid driven square cavity with differentially heated sidewalls has not been examined thus far, although, as mentioned earlier, it is of importance in several processes. Results of a simulation are presented here through a new numerical method which is described in full in later sections.

Thus, the main aim of this study is to simulate the mixed convection flow of a Bingham fluid in a differentially heated lid-driven cavity, using the Bingham and Papanastasiou models, employing TFDDFM. The relevant equations are listed in Section 3, and the numerical method is described in Section 4. In Section 5, the present results are validated with previous numerical investigations and, in Section 6, the effects of the main parameters, viz., the Reynolds number, the Prandtl number and the Bingham number on the flow and thermal fields are exhibited, along with the location and shapes of various yield surfaces. Comparison with the results obtained by using the Bingham model is also provided.

2. Bingham and Papanastasiou models

The constitutive equation for an incompressible Bingham fluid is based on the assumption that the fluid remains at rest or moves as a rigid body if the second invariant of the extra stress tensor $\boldsymbol{\tau}$ is less than or equal to the yield stress τ_y. If the second invariant exceeds the yield stress, the material flows like a fluid. This second invariant is defined through

$$II(\boldsymbol{\tau}) = (1/\sqrt{2})\sqrt{\boldsymbol{\tau} : \boldsymbol{\tau}}. \tag{2.1}$$

Hence, using the first Rivlin–Ericksen tensor $\mathbf{A}_1$ (Rivlin and Ericksen [25]), the rigidity condition is given by

$$\mathbf{A}_1 = \mathbf{0}, \quad II(\boldsymbol{\tau}) \leqslant \tau_y. \tag{2.2}$$

When the magnitude of the extra stress tensor exceeds the yield stress, one defines $\boldsymbol{\tau}$ as a function of the tensor $\mathbf{A}_1$ leading to the following relation:

$$\boldsymbol{\tau} = \eta\mathbf{A}_1 + \frac{\tau_y}{II(\mathbf{A}_1)}\mathbf{A}_1, \quad II(\boldsymbol{\tau}) > \tau_y. \tag{2.3}$$

It is well known that the absence of a constitutive relation which applies throughout the flow has led to approximate models such as the Papanastasiou and the bi-viscosity models. In the Papanastasiou model (Papanastasiou [8]), which is of interest here, the constitutive equation for the incompressible Bingham fluid is replaced by that of a material with a non-Newtonian viscosity. That is,

$$\boldsymbol{\tau} = \eta(II(\mathbf{A}_1))\mathbf{A}_1, \tag{2.4}$$

where the viscosity η is the sum of the constant Newtonian viscosity η_0, and a parameter dependent term. To be specific,

$$\eta(II(\mathbf{A}_1)) = \eta_0 + \frac{\tau_y}{II(\mathbf{A}_1)}[1 - \exp(-mII(\mathbf{A}_1))], \tag{2.5}$$

where $m > 0$ is the parameter which has to be chosen. Note that the viscosity function in Eq. (2.5) is a smooth function of its argument. As far as numerical modelling is concerned, one can employ Eq. (2.5) and choose an appropriate value for the parameter m. A search through the literature shows that m can be as large as 10^6. Here, we examine the consequences of varying m from 100 to 1000; see Section 6 below.