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A model for an application to biomedical engineering through nanoparticles

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ABSTRACT

Recent advancements in nanoscience and technology has made the nanofluid an important research topic. Various models have been put forward to estimate the effective thermal conductivity of nanofluids. Present article addresses the comparative study of Maxwell's and Hamilton–Crosser's model for mixed convection peristaltic flow of incompressible nanofluid in an asymmetric channel. Viscous dissipation and heat generation/absorption effects are retained. Analysis is performed for five different types of nanoparticles namely titanium oxide or titania (TiO_2), aluminum oxide or alumina (Al_2O_3), copper oxide (CuO), copper (Cu) and silver (Ag) with water as base fluid. Velocity and thermal slip conditions are employed. Lubrication approach is adopted for problem formulation. The developed non-linear problems are solved numerically. Plots for axial velocity, temperature and heat transfer rate at the wall are obtained and analyzed.

1. Introduction

Heat transfer enhancement determines the need for innovative coolants with improved performance. The novel concept of nanofluid has been introduced to enhance the heat transfer capability of conventional coolants such as water, ethylene glycol and oils. The terminology of nanofluid was first used by Choi [1] who demonstrated an anomalous increase in thermal conductivity of water and other liquids through dispersion of copper and aluminum nanoparticles. This enhanced feature of nanofluid has led to the plethora of diverse applications such as cooling of microelectronics, engine cooling/vehicle thermal management, magnetic drug targeting, space cooling, in grinding machining and many others. Utilization of nanofluids in water cooled nuclear reactor can produce safety margins and provides economic gains. Nanofluids are also employed in solar collectors for their tunable optical properties. Nanofluids can improve the performance efficiency of heat exchangers by reducing total heat resistance. In biomedicine, nanofluids serve as carriers for delivering drugs and radiation in cancer patients. Nanofluids can be considered as single phase liquids and hence classical theory of single phase fluids can be applied in which physical properties of nanofluids are expressed as function of both constituents and their volume fractions. Keeping this in view Tiwari and Das [2] proposed a mathematical model to estimate heat transfer behavior by varying nanoparticle volume fraction. Buongiorno [3] also suggested a mathematical model for nanofluids convective transport considering novel aspects of Brownian motion and thermophoresis. Both the above mentioned models have been applied to investigate heat transfer problems involving nanofluids [4–15].

Peristaltic flow refers to the transportation of fluid inside a channel or tube by the action of flexible walls. It is a major mechanism for fluid flow in many biological and industrial systems. Within human body it is involved in swallowing of food through esophagus, movement of chyme in the gastro-intestinal track, in the ductus efferentes of the male reproductive system, vasomotion of small blood vessels such as arterioles and capillaries. Peristaltic pumps are used to transport corrosive materials in order to avoid direct contact of the fluid with the pump's internal surface. Many biomedical devices as dialysis machines, open heart bypass pump machines, infusion pumps, etc. are engineered on the mechanism of peristaltic. In robotic industry, the concept of peristalsis has been used for the production of robots. The pioneering work on peristalsis was performed to investigate the urine transport. Shapiro [16] in 1967 analyzed the peristaltic pumping in a two dimensional flexible tube. Later, the theoretical results determined in [16] were experimentally confirmed by Weinberg [17]. Some

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Nomenclature

$\overline{H_1}, \overline{H_2}$	right and left walls
$(\overline{X}, \overline{Y})$	space coordinates in fixed frame
K_f	thermal conductivity of base fluid
$(\overline{U}, \overline{V})$	velocity components in fixed frame
K_p	thermal conductivity of nanoparticle
K_{eff}	effective thermal conductivity
a_1, b_1	dimensional wave amplitude
γ	phase difference
F	dimensionless flow rate in wave frame
η	dimensionless flow rate in fixed frame
$\overline{Q}$	volume flow rate in fixed frame
$\overline{q}$	volume flow rate in wave frame
h_1, h_2	dimensionless right/left walls
Φ	heat generation/absorption
T_0, T_1	temperature of right/left wall
ρ_{eff}	effective density of nanofluid
μ_{eff}	effective viscosity of nanofluid
$(\rho C)_{eff}$	effective heat capacity of nanofluid
$(\rho\beta)_{eff}$	effective thermal expansion of nanofluid
ε	dimensionless heat generation/absorption
ξ	dimensionless velocity slip parameter
n	shape factor of nanoparticles
ρ_f, ρ_p	density of fluid and nanoparticle
β_f	fluid thermal expansion coefficient
β_p	nanoparticle thermal expansion coefficient
a, b	dimensionless wave amplitudes
$\overline{t}$	time in fixed frame
$d_1 + d_2$	width of channel
ϕ	nanoparticle volume fraction
λ	wavelength
$(\overline{x}, \overline{y})$	space coordinates in wave frame
$(\overline{u}, \overline{v})$	velocity components in wave frame
$\overline{p}$	pressure in wave frame
(x, y)	dimensionless space coordinates
$\overline{P}$	pressure in fixed frame
T_m	mean temperature
c	wave speed
T	fluid temperature
g	gravity
ψ	stream function
Pr	Prandtl number
Br	Brinkman number
Re	Reynolds number
δ	wave number
Ec	Eckert number
Gr	Grashoff number
ζ	thermal slip parameter
C_f	specific heat of fluid
μ_f	dynamic viscosity of fluid
C_p	specific heat of nanoparticle
d	ratio of d_2 and d_1

recent researches dealing with the peristaltic motion are mentioned in Refs. [18–27].

The effect of heat transfer cannot be ignored in peristalsis especially when dealing with the blood flow simulation related to tumors and muscles, drug transport, production of osteoinductive material, nutrients to brain cell, etc. Peristalsis is also seen in the processes of oxygenation and hemodialysis. Some references pertaining to the heat transfer phenomenon in peristalsis are mentioned in [28–37]. Peristaltic transport of nanofluids are utilized in modern drug delivery systems and in cancer therapy to destroy undesirable tissues. Despite the aforementioned applications, very little attention has been given to the studies dealing with the peristaltic transport of nanofluids [38–42].

Current study aims to address the effects of velocity and thermal slip on the peristaltic transport of nanofluids through an asymmetric channel. Two different models for effective thermal conductivity of nanofluids are utilized. Long wavelength and low Reynolds number approximation are adopted for problem formation. Arising system comprising of coupled equations has been solved numerically through NDSolve of MATHEMATICA. Graphs are sketched to see the effects of embedded parameters on velocity and temperature distributions.

2. Problem formulation

We consider the peristaltic motion of an incompressible nanofluid flowing through a two dimensional vertical asymmetric channel of width $d_1 + d_2$ (see Fig. 1). Nanofluid comprises of five different types of nanoparticles namely titanium oxide or titania (TiO_2), aluminum oxide or alumina (Al_2O_3), copper oxide (CuO), copper (Cu) and silver (Ag). Water is considered as the base fluid. We take into account two different models for effective conductivity of nanofluids. Maxwell–Garnelt model for effective thermal conductivity is expressed as [43]:

$$\frac{K_{eff}}{K_f} = \frac{K_p + 2K_f - 2\phi(K_f - K_p)}{K_p + 2K_f + \phi(K_f - K_p)}. \tag{1}$$

Hamilton–Crosser model the thermal conductivity of nanofluid is given by [44]:

$$\frac{K_{eff}}{K_f} = \frac{K_p + (n-1)K_f - (n-1)\phi(K_f - K_p)}{K_p + (n-1)K_f + \phi(K_f - K_p)}. \tag{2}$$

Here K_{eff} is the effective thermal conductivity of the nanofluid, K_f is the thermal conductivity of water, K_p is the thermal conductivity of nanoparticles and ϕ is the nanoparticle volume fraction. n in

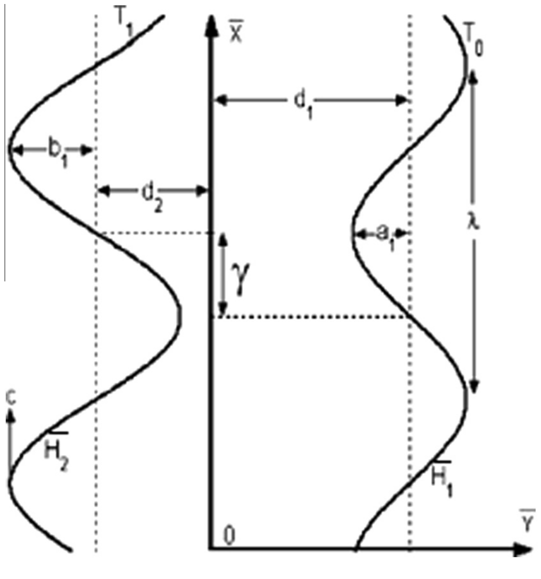

Fig. 1. Geometry of the problem.