International Journal of Heat and Mass Transfer 102 (2016) 585–595

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Effects of velocity slip and temperature jump on the heat transfer and entropy generation in micro porous channels under magnetic field

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A R T I C L E   I N F O

Article history:
Received 2 April 2016
Received in revised form 23 June 2016
Accepted 23 June 2016

Keywords:
Microchannel with thick walls
Second law of thermodynamics
Temperature jump
Slip velocity
Nusselt number

A B S T R A C T

A thermodynamic analysis was performed for thick walled microchannels using the velocity slip and temperature jump at the interface of the solid–fluid phases. The slip velocity and temperature jump effects were assumed to be equal for both the upper and lower walls. A Darcy–Brinkman model was used in the application of the momentum equation in the porous section of the channel and the effects of the magnetic field were considered. The thermal boundary conditions for two separate cases were proposed; case one assumed a constant high temperature for the lower wall and a constant low temperature for the upper wall, and case two which assumed constant heat flux boundary conditions for the lower wall and convective heat transfer for the upper wall. Constant, but different, internal heat generations were incorporated into the energy equations for all three parts of the system, and a combined analytical–numerical solution procedure was then applied. A comprehensive investigation of the effects of the slip velocity and temperature jump on the velocity field, temperature distribution, Nusselt number, and local entropy generation rate was performed. The results indicate that a variation on the magnetic field may change the fluid velocity at the solid–fluid interface. Interestingly, it was shown that by using a specific value for the temperature jump coefficient and variations in the thickness of the upper and/or lower wall thickness, it may be possible to achieve the maximum Nusselt number. However, depending on the value of the temperature jump, the Nusselt number may continuously increase or decrease, depending on the wall thickness.

1. Introduction

The rapid development and deployment of advanced manufacturing techniques has resulted in a significant reduction in the size of the individual components within many devices. Among these, are thermal systems which involve heat transfer and fluid flow. These reductions in size, have led to the development of a number of experimental, numerical and analytical studies that can be employed to characterize and visualize the temperature fields in these systems [1–3]. Previous investigations have indicated that these types of systems may be influenced by various parameters, the most important of which are the velocity slip and temperature jump, occurring at the solid–liquid interface in small scale systems [1,4–6]. These two parameters have been shown to have a significant impact on the flow and temperature fields, which can ultimately impact the thermal performance of the system. More recently, attention has been focused on trying to better understand the impact of these parameters on various types of thermal systems [7,8]. It is important to note here, that the slip velocity and temperature jump are also apparent in macro-scale devices [9–11]. For example, wall slip is an important parameter in the polymer industry because of the instabilities occurring at high stress levels [10]. Although not directly related, the effect of the temperature jump on the thermal analyses of systems is clearly apparent, even in the thermal analyses of conductive small scale components [12]. With respect to micro devices, the temperature jump and velocity slip become increasingly important when the Knudsen number (Kn), defined as the ratio of the mean free path to the fluid system size, is large. It has been proposed that for Knudsen numbers higher that 0.01 the continuum assumption used in the derivation of the Navier–Stokes equations breaks down [4,13], and hence the temperature jump and slip velocity boundary conditions should be assumed. If in fact this is the case, the continuum assumption can be assumed and the Kn should be below 0.01 in order for the non-slip boundary conditions to be valid [14].

A number of investigations have been conducted to examine the effect of the slip velocity and temperature jump in these systems. One of the early investigations was conducted by Khadrawi

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.06.080

Nomenclature

a_1	slope of the thermal conductivity versus temperature for lower solid material, K^{-1}
a_2	slope of the thermal conductivity versus temperature for upper solid material, K^{-1}
B_0	magnetic field, T
Br	Brinkman number ($\mathrm{Pr} \cdot \mathrm{Ec}$)
Da	Darcy number
Ec	Eckert number
h	convection heat transfer (Case two), $W \cdot m^{-2} \cdot K^{-1}$
h_3	height of the channel, m
k_1	reference thermal conductivity for lower solid material, $W \cdot m^{-1} \cdot K^{-1}$
k_2	reference thermal conductivity for upper solid material, $W \cdot m^{-1} \cdot K^{-1}$
k_{eff}	effective thermal conductivity of porous medium, $W \cdot m^{-1} \cdot K^{-1}$
k_{e1}	ratio of porous medium thermal conductivity to lower solid material thermal conductivity
k_{e2}	ratio of porous medium thermal conductivity to upper solid material thermal conductivity
N_s	dimensionless local entropy generation rate
Nc	dimensionless convection heat transfer (Case two)
M	Hartmann number
Pr	Prandtl number
Q_1	dimensionless volumetric internal heat generation rate for the lower solid material
Q_2	dimensionless volumetric internal heat generation rate for the upper solid material
Q_H	dimensionless heat flux boundary condition (Case two)
Q_p	dimensionless volumetric internal heat generation rate for the porous medium
$\dot{q}_1$	volumetric internal heat generation rate for the lower solid material, $W \cdot m^{-3}$
$\dot{q}_2$	volumetric internal heat generation rate for the upper solid material, $W \cdot m^{-3}$
q_H	heat flux boundary condition (Case two), $W \cdot m^{-2}$
$\dot{q}_p$	volumetric internal heat generation rate for the porous medium, $W \cdot m^{-3}$
$\dot{S}'''$	local entropy generation rate, $W \cdot m^{-3} \cdot K^{-1}$
T	temperature, K
T_1	temperature of the lower solid material, K
T_2	temperature of the upper solid material, K
T_C	outer temperature of the upper solid material, K
T_H	inner temperature of the lower solid material, K
T_p	temperature of the porous medium, K
U	dimensionless velocity
u	velocity of the fluid in porous medium, $m \cdot s^{-1}$
u_r	characteristics velocity

Greek symbols

α	velocity slip coefficient
α_1	dimensionless slope of the thermal conductivity versus temperature for lower solid material
α_2	dimensionless slope of the thermal conductivity versus temperature for upper solid material
β	temperature jump coefficient
κ	permeability, m^2
ε	porosity
μ_f	dynamic viscosity of the base fluid, $kg \cdot s^{-1} \cdot m^{-1}$
θ	dimensionless temperature
θ_1	dimensionless temperature of the lower solid material
θ_2	dimensionless temperature of the upper solid material
θ_p	dimensionless temperature of the porous medium
θ_H	dimensionless temperature at outer side of the lower wall
γ	specific heat ratio, C_P/C_V
λ	mean free path, m
σ	electrical conductivity of fluid, $S \cdot m^{-1}$
σ_V	momentum accommodation coefficient
σ_T	thermal accommodation coefficient

and Al-Shyyab [15], in which the problem of heat and fluid flow for axially moving micro-concentric cylinders, with the consideration of both slip velocity and temperature jump was addressed. Due to the absence of nonlinearities, an analytical solution for the temperature distribution was proposed. Chen and Tian [13] studied the fluid flow and heat transfer between two horizontal parallel plates. In this investigation, a lattice Boltzmann numerical technique was used and Langmuir model for the velocity slip and temperature jump was implemented. The investigation proposed the use of the Langmuir slip model as an alternative for the well-known Maxwell slip model. Other early investigations on the effect of slip velocity and temperature jump on microtubes were performed by Aziz and Niedbalski [16], who analyzed the multi-dimensional energy equation for microtubes with respect to both radial and axial coordinates. A finite difference technique was used to obtain a solution, and the velocity in the central region of the microtube was found to be higher when using the second-order slip model than when the first-order model was used. Moreover, it was concluded that the local Nusselt number (Nu) decreases as Kn increased. Malvandi and Ganji [17] considered the problem of fluid flow and heat transfer in a vertical microchannel, using water/alumina nanofluids. In this investigation, the mixed convection and slip velocity assumptions were utilized in the formulation and also included the nanoparticles fraction equation in the investigation. The results indicated that the nanoparticle size has a significant effect on the distribution of the nanoparticles and the heat transfer rate.

Jha and Aina [18] used first-order models for both slip velocity and temperature jump boundary conditions to analyze the fully developed mixed convection flow and heat transfer of incompressible and viscous fluids in a vertical micro porous annulus. The velocity and temperature fields together with the Nu for a number of Prandtl and Kn combinations and both injection/suction parameter values were investigated. The results agreed quite well with previous findings, and demonstrated that the maximum velocity decrease as Kn increases.

Unlike the first law of thermodynamics, which is silent about the quality of a thermal process, the second law provides qualitative information about a system [19]. This law, which uses both temperature and heat flux formulations, can be utilized to analyze and optimize a system from a quality perspective [19,20]. Using the entropy concept, important information regarding the irreversibility of the system and exergy destruction can be determined. Surprisingly, there have been few investigations that utilize the Second Law in the analysis of microchannel systems. Chen and Tian [21] extended the study of fluid flow and heat transfer between two horizontal parallel plates, which used a Langmuir slip model for the velocity and temperature fields [13] with an entropy generation analysis [21]. Both local and total entropy generation rates were reported and illustrated. For some specific