International Journal of Heat and Mass Transfer 101 (2016) 156–165

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Peristaltic transport of Powell–Eyring fluid in a curved channel with heat/mass transfer and wall properties

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ARTICLE INFO

Article history:
Received 31 December 2015
Received in revised form 7 May 2016
Accepted 7 May 2016
Available online 26 May 2016

Keywords:
Peristalsis
Curved channel
Powell–Eyring fluid
Complaint wall
Analytic solution

ABSTRACT

In this work, we explore the peristaltic flow of Powell–Eyring fluid through curved passage with complaint walls. Heat and mass transfer analyses in the presence of viscous dissipation and thermophoresis effects are performed. Adequate assumptions of long wavelength and low Reynolds number are accounted for problem formulation. The arising system of partial differential equations is solved by using regular perturbation method. Our results show that the material parameters of the Powell–Eyring fluid strongly affect the flow fields. We observe that symmetry in the profiles is not preserved in the curved channel. The size and shape of the trapped bolus is different in the upper and lower halves of the curved channel. A comparative study of curved and planar channels is also presented.

1. Introduction

The process of area contraction and expansion along the length of a distensible tube/channel is known as peristalsis. This type of phenomenon appears in many physiological processes such as urine flow from kidney to bladder, the flow of food bolus through gastrointestinal tract, transport of chyme through small intestine etc. Peristaltic mechanism occurs in designing some biomedical instruments including heart lung machine and blood pumps. In industry, this phenomenon appears in the transport of toxic fluid. On the other hand, many industrial and biological fluids such as polymers, liquid detergents, slurries, shampoos, toothpastes, fruit juices, gypsum pastes, printer inks, blood, multi-grade oils etc. are characterized by the non-linear relationship between stress and deformation rate and are termed as non-Newtonian. Peristaltic transport of non-Newtonian fluids has been given significant attention by the research community. For instance, Abd elmaboud and Mekheimer [1] investigated the peristalsis of second order fluid occupying a porous space. They adopted perturbation approach to construct series solutions of the arising non-linear problem. MHD peristaltic transport of couple-stress fluid was examined by Tripathi and Beg [2]. Their study was motivated towards the blood flow in microcirculatory system with particle size effect. Numerical solutions for peristaltic motion of Carreau–Yasuda fluid in a curved channel have been provided by Hayat et al. [3]. Kothandapani et al. [4] analytically discussed the peristaltic motion of Carreau fluid through tapered asymmetric channel. Kothandapani and Prakash [5] examined the peristaltic flow of Williamson nanofluid considering the novel Brownian motion and thermophoresis effects. MHD Peristaltic flow of fourth grade fluid in a rotating channel was addressed by Abd-Alla et al. [6]. They found that magnetic field and rotational effects tend to oppose the fluid motion. Mustafa et al. [7] analyzed the peristaltic motion of fourth-grade fluid through a vertical channel using Keller-box method. Further recent studies in this direction can be found in Refs. [8–13].

The commonly discussed power-law fluid model has tendency to describe shear-thinning as well as shear-thickening effects. The former is a common characteristic of many non-Newtonian fluids which include blood, polymers and composite materials. The Powell–Eyring fluid model [14] is more advantageous than the power-law fluid model in the sense that its constitutive equations have been derived from the kinetic theory of gases rather than the empirical relationships. Further it can accurately reduce to the Newtonian flow behavior for low and high shear rates. Peristalsis through Powell–Eyring fluid model has been discussed

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.05.034
0017-9310/

by some recent researchers. For example, Akbar and Nadeem [15] discussed the effects of heat and mass transfer on the peristaltic flow of Eyring–Powell fluid. Hayat et al. [16] investigated the slip effects on the peristaltic flow of Eyring–Powell fluid. Abbasi et al. [17] discussed the peristaltic flow of Eyring–Powell fluid in a curved channel. Hayat et al. [18] analyzed the effects of convective conditions and chemical reaction on peristaltic flow of Eyring–Powell fluid with wall properties. Very recently, analytic solutions for MHD peristaltic motion of Powell–Eyring fluid through a planar channel with wall properties and viscous dissipation were provided by Hina et al. [19].

The study of peristalsis with the consideration of wall properties has special significance in processes such as blood flow in arteries and veins, urine flow in the urethras and air flow in the lungs. In the past, peristaltic flows under the influence of wall properties have been discussed by various researchers. For example, Radhakrishnamacharya and Srinivasulu [20] investigated the heat transfer effects in peristaltic flow of Newtonian fluid with wall properties. Muthu et al. [21] analyzed the peristaltic motion of micropolar fluid in circular cylindrical tubes having complaint walls. Later, Hayat et al. [22,23] investigated the influence of wall properties on the MHD peristaltic flows of Jeffery fluid and Johnson–Segalman fluid respectively. Series solutions valid for small non-Newtonian fluid parameters were presented. Srinivas and Kothandapani [24] described the characteristics of heat and mass transfer in MHD peristaltic flow through a porous channel with wall properties. Peristaltic flow of Prandtl fluid in rectangular duct characterized by complaint walls was addressed by Riaz et al. [25]. Peristaltic motion of Burgers' fluid with wall properties was explored by Javed et al. [26]. Combined influence of Hall current and complaint walls on peristalsis were examined by Gad et al. [27]. Hina et al. [28] presented an analytical study for peristaltic flow of shear-thinning fluid in curved channel subject to wall properties.

To our knowledge, peristalsis of Powell–Eyring fluid through curved channel with complaint walls has not been explored previously. Thus present work is undertaken to fill this void in the presence of heat and mass transfer effects. The effects of viscous dissipation and thermophoresis are considered in the transport equations. Series expressions of stream function, temperature and concentration are developed. Graphical results are obtained to explain the effects of parameters entering in the problem.

2. Mathematical formulation

Consider the peristaltic motion of Powell–Eyring fluid in a curved channel in the presence of heat and mass transfer. The channel of width $2d_1$ is coiled in a circle of center C and radius R^*. Flow in caused due to the sinusoidal waves traveling across the channel walls. These waves possess neuromuscular properties of any tubular smooth muscle. u and v denote the axial and radial components of velocity. The wave shapes are expressed as

$$r = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right], \tag{1}$$

in which c is the wave speed, λ is the wavelength and a is the wave amplitude.

The stress tensor for Powell–Eyring fluid is given by

$$\tau = \left[\mu + \frac{1}{\beta\dot{\gamma}}\sinh^{-1}\left(\frac{1}{c_1}\dot{\gamma}\right)\right]\mathbf{A}_1, \tag{2}$$

where

$$\dot{\gamma} = \sqrt{\frac{1}{2}tr(\mathbf{A}_1)^2}, \quad A_1 = grad\mathbf{V} + (grad\mathbf{V})^T. \tag{3}$$

Here μ is the dynamic viscosity, β and c_1 are the material fluid parameters. We consider the expansion of $\sinh^{-1}$ up to second order as

$$\sinh^{-1}\left(\frac{1}{c_1}\dot{\gamma}\right) \cong \frac{\dot{\gamma}}{c_1} - \frac{\dot{\gamma}^3}{6c_1^3}, \quad \frac{\dot{\gamma}^5}{c_1^5} << 1. \tag{4}$$

The flow problem is governed by the following equations:

$$\frac{\partial v}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial u}{\partial x} + \frac{v}{r+R^*} = 0, \tag{5}$$

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial v}{\partial x} - \frac{u^2}{r+R^*}\right] = -\frac{\partial p}{\partial r} + \frac{1}{r+R^*}\frac{\partial}{\partial r}\{(r+R^*)\tau_{rr}\} + \frac{R^*}{r+R^*}\frac{\partial \tau_{xr}}{\partial x} - \frac{\tau_{xx}}{r+R^*}, \tag{6}$$

$$\rho\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial u}{\partial x} + \frac{uv}{r+R^*}\right] = \frac{1}{(r+R^*)^2}\frac{\partial}{\partial r}\left\{(r+R^*)^2\tau_{rx}\right\} + \frac{R^*}{r+R^*}\frac{\partial \tau_{xx}}{\partial x} - \frac{R^*}{r+R^*}\frac{\partial p}{\partial x}, \tag{7}$$

$$\rho C_p\left[\frac{\partial}{\partial t} + v\frac{\partial}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial}{\partial x}\right]T = \kappa\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right] + (\tau_{rr} - \tau_{xx})\frac{\partial v}{\partial r} + \tau_{xr}\left(\frac{\partial u}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial v}{\partial x} - \frac{u}{r+R^*}\right), \tag{8}$$

$$\left[\frac{\partial}{\partial t} + v\frac{\partial}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial}{\partial x}\right]C = D\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\right]C + \frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right]. \tag{9}$$

$$\boldsymbol{\tau} = \left[\mu + \frac{1}{\beta c_1}\left(1 - \frac{1}{6c_1^2}\left\{\begin{array}{c} 2\left(\frac{\partial v}{\partial r}\right)^2 + 2\left(\frac{R^*}{r+R^*}\frac{\partial u}{\partial x} + \frac{v}{r+R^*}\right)^2 \\ +\left(\frac{\partial u}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial v}{\partial x} - \frac{u}{r+R^*}\right)^2 \end{array}\right\}\right)\right] \times \begin{bmatrix} 2\frac{\partial v}{\partial r} & \frac{\partial u}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial v}{\partial x} - \frac{u}{r+R^*} \\ \frac{\partial u}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial v}{\partial x} - \frac{u}{r+R^*} & 2\left(\frac{R^*}{r+R^*}\frac{\partial u}{\partial x} + \frac{v}{r+R^*}\right) \end{bmatrix}. \tag{10}$$

The boundary conditions for the present problem are

$$u = 0, \quad T = \begin{Bmatrix} T_1 \\ T_0 \end{Bmatrix}, \quad C = \begin{Bmatrix} C_1 \\ C_0 \end{Bmatrix} \text{ at } r = \pm\eta, \tag{11}$$

$$R^*\left[-\tau\frac{\partial^3}{\partial x^3} + m\frac{\partial^3}{\partial x\partial t^2} + d\frac{\partial^2}{\partial t\partial x}\right]\eta = \frac{1}{(r+R^*)}\frac{\partial}{\partial r}\left\{(r+R^*)^2\tau_{rx}\right\} + R^*\frac{\partial \tau_{xx}}{\partial x} - \rho(r+R^*)\times\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial u}{\partial x} + \frac{uv}{r+R^*}\right] \text{ at } r = \pm\eta, \tag{12}$$

where p is the pressure, ρ the density, C_p the specific heat, κ the thermal conductivity, D the diffusion coefficient of the diffusing species, T_m the mean fluid temperature, K_T the thermal diffusion ratio, T and C denote the fluid temperature and concentration, τ the elastic tension, m the mass per unit area, d the coefficient of viscous damping, τ_{xr}, τ_{rr} and τ_{xx} are the components of the stress tensor. We now introduce the following non-dimensional variables: