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Thermoelastic damping in micro- and nanomechanical beam resonators considering size effects

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ABSTRACT

In this paper, we describe governing equations for modified coupled thermoelasticity in micro- and nanomechanical beam resonators, which can treat the effects of size by taking the relaxation time, the phonon mean-free path and the material length scale parameter into account. An analytical model of thermoelastic damping is derived using the complex-frequency approach. Numerical results of thermoelastic damping calculated using the proposed model are presented and compared to those calculated using the models proposed by Zener and Lifshitz and Roukes for a silicon thin beam. The results show that the nonlocal effect characterized by the length scale parameter is negligible even at sizes down to the phonon mean-free path. The size effects characterized by the relaxation time and the phonon mean-free path are significant for a micron-scale beam. Device miniaturization beyond the submicron scale will lead to increased energy dissipation due to thermoelastic damping considering size effects. The influence of size effects on thermoelastic damping can be suppressed by increasing aspect ratios. Finally, we present the range of geometry of a silicon beam resonator where the effects of size can be neglected by taking 10% as the permitted error bound.

1. Introduction

Micro- and nanomechanical resonators are widely used as sensors, microwave transceivers and modulators depending on their resonant modes [1–3]. In most applications, it is important to have design control over loss of energy for designing high-performance components. Among different energy dissipation mechanisms in these resonators, thermoelastic damping has been identified as a significant loss mechanism at room temperature in vacuum through both experimental and theoretical studies [4–8], in which energy is dissipated due to irreversible heat conduction. Accurate analysis of thermoelastic damping is crucial for designing low-loss micro- and nanomechanical resonators.

The amount of thermoelastic damping is usually expressed in terms of the inverse of the quality factor Q. In 1937, Zener provided a theoretical foundation for thermoelastic damping and developed a simple expression to calculate Quality factor for a vibrating beam in its flexural mode [9,10]. Zener's classical formula is:

$$Q^{-1} = \frac{E\alpha^2 T_0}{\rho C_e}\frac{\omega_n \tau}{1+\omega_n^2\tau^2} = \Delta_z \frac{\Omega}{1+\Omega^2} \tag{1}$$

where E is Young's modulus, α is the linear coefficient of thermal expansion, T_0 is the equilibrium temperature, ρ is the density, C_e is the specific heat, ω_n ($n = 1, 2,\ldots$) denotes the undamped natural frequency of the nth mode of flexural vibration, τ is a time constant, $\Delta_z = \frac{E_{ad}-E}{E} = \frac{E\alpha^2 T_0}{\rho C_e}$ is the Zener modulus, E_{ad} is the adiabatic value of Young's modulus, E is the isothermal value of Young's modulus, and $\Omega = \omega_n\tau$ is the normalized frequency.

Lifshitz and Roukes improved upon Zener's formula by developing an exact expression for thermoelastic damping in a thin rectangular beam [11]. The formula of Lifshitz and Roukes is given by:

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right| = \frac{6T_0 E\alpha^2}{\rho C_e \xi^2}\left(1 - \frac{1}{\xi}\frac{\sinh\xi + \sin\xi}{\cosh\xi + \cos\xi}\right), \tag{2}$$

where $\xi = h\sqrt{\frac{\omega_n \rho C_e}{2\lambda}}$, λ is the thermal conductivity and h is the thickness.

The analytical models of Zener and Lifshitz and Roukes are now widely used to estimate thermoelastic damping in a beam.

Size compatibility with monolithic integrated electronic circuits requires orders-of-magnitude reduction in the dimensions of resonators and minimization of the damping [12,13]. The present dimensions of micro- and nanomechanical resonators can range from 1 μm to 1000 μm in length, typically, and from a few μm down to 25 nm or even sub-nm regimes in thickness or diameter

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[14], which overlap with the phonon mean free path and the material length scale parameter. The small dimensions of these resonators pose a serious challenge as size effects on material properties and behavior at the scale of microns have been shown experimentally in recent years. The analytical models of Zener and Lifshitz and Roukes were obtained based on classical linear thermoelastic theory and size effects were neglected. Thus, the questions arise: what is the valid range of geometry of micro- and nanomechanical resonators for these two models, and how to evaluate the thermoelastic damping if these two models are invalid.

Size effects will affect thermal transport and strain energy in nanostructures. When the thickness of the beam is typically of the order of microns or nanometers, the size effects are often observed experimentally not only in the area of thermal transport, but at the area of elasticity theory. Thermal transport in silicon crystals is mediated by phonons. If the characteristic length scale of a silicon beam is much larger than the phonon mean-free path, the classical heat diffusion equation can describe accurately the transport of thermal energy at a large time scale. However, in nanostructures, the Fourier equation is not a rigorous description for the temperature gradient and the heat current becomes nonlocal because the length and time scales of interest overlap with the mean-free path and the relaxation time of phonons [15,16]. Also, classical Bernoulli–Euler beam model cannot capture the size effect and the classical couple stress elasticity theory should be modified by introducing an internal material length scale parameter [17]. That is, size effects on thermoelastic damping at the scale of microns should be analyzed from the modified coupled thermoelastic equations. To do that, non-Fourier thermal conduction should be involved in the heat transfer equation to consider the finite velocity of heat transfer. Moreover, nonlocal effects characterized by the phonon mean-free path and the material length scale parameter should be considered in the heat transfer equation and in the equation of motion.

In this paper, the modified coupled thermoelastic equations in a thin beam considering size effects are described. The analytical model of thermoelastic damping is derived using the complex-frequency approach based on the governing equations obtained. The influence of size effects on thermoelastic damping is studied. The difference between the results of thermoelastic damping calculated using the proposed model with those calculated using the analytical models of Zener and Lifshitz and Roukes is investigated. The range of geometry of micro- and nanomechanical silicon resonators where the effects of size can be neglected is discussed.

2. Governing equations of modified coupled thermoelasticity in a thin beam considering size effects

In this section, we describe the coupled thermoelastic equations in a thin beam considering size effects. Some simplifications and assumptions are used and listed here.

(1) The Euler–Bernoulli beam theory is used to deduce the governing equations; thus, only thin beams undergoing small flexural deformations are considered. Large amplitude deformations where the Euler–Bernoulli theory is known to fail are not discussed in this paper.
(2) All of the material properties are constants and a small temperature increment is considered.
(3) Thermal gradients in the plane of the cross section along the thickness direction are much larger than gradients along the beam axis, and no gradients exist in the width direction. Also, there is no heat flow across the boundaries of the beam.

A thin cantilever beam of length L, thickness b, and width c is considered. A Cartesian coordinate system is attached to the beam so that the x-coordinate is parallel to the axis of the beam. The y- and z-axes are parallel to the thickness and width directions, respectively. The beam undergoes bending vibrations of small amplitude about the x-axis. The initial condition is that the beam is unstressed, unstrained and at temperature T_0 over its entirety.

Now we deduce the governing equations considering the size effects. Non-Fourier thermal conduction and nonlocal effects characterized by the phonon mean-free path are involved in the heat transfer equation. The material length scale parameter is involved in the equation of motion. Thus, the governing equations in [11] are extended to the scale of microns by introducing the relaxation time, the phonon mean-free path and the material length scale parameter.

The energy equations for an elastic object subject to small deformations can be expressed as:

$$\rho\dot{e} = \rho T\dot{s} + \sigma_{ij}\dot{\varepsilon}_{ij} \tag{3}$$

and

$$\rho T\dot{s} = \rho r - div(q_i), \tag{4}$$

where T is temperature, σ_{ij} is stress component, ε_{ij} is strain component, e is the internal energy density, s is the entropy density, q_i is the heat flux vector and r is the internal heat source density.

We can obtain the following equations by introducing the Helmholtz free-energy function $\phi(\varepsilon_{ij}, T)$ defined as $\phi(\varepsilon_{ij}, T) = e(\varepsilon_{ij}, T) - Ts(\varepsilon_{ij}, T)$:

$$\frac{\partial}{\partial t}\phi(\varepsilon_{ij}, T) = \frac{\partial\phi}{\partial\varepsilon_{ij}}\frac{\partial\varepsilon_{ij}}{\partial t} + \frac{\partial\phi}{\partial T}\frac{\partial T}{\partial t} = \frac{\partial e}{\partial t} - \frac{\partial(Ts)}{\partial t},$$
$$\sigma_{ij} = \rho\frac{\partial\phi}{\partial\varepsilon_{ij}} \quad \text{and} \quad s = -\frac{\partial\phi}{\partial T}. \tag{5}$$

Assuming that the specific heat $C_e = -T\frac{\partial^2\phi}{\partial T^2}$, we have

$$div(q_i) = \rho r - \rho C_e\frac{\partial T}{\partial t} + T\frac{\partial\sigma_{ij}}{\partial T}\frac{\partial\varepsilon_{ij}}{\partial t}, \tag{6}$$

The linear constitutive equation is given by:

$$\sigma_{ij} = \lambda\varepsilon\delta_{ij} + 2G\varepsilon_{ij} - \frac{E\alpha\theta}{1-2\mu}\delta_{ij}, \tag{7}$$

where δ_{ij} is the Dirac function, λ and G are the Lamé coefficients, μ is the Poisson's ratio, and $\theta = T - T_0$ is the temperature increment.

Form the Euler–Bernoulli beam theory, the axial stress and the axial strain in the thin beam meet:

$$\varepsilon_{xx} = -y\frac{\partial^2 w(x,t)}{\partial x^2} \quad \text{and} \quad \sigma_{xx} = -Ey\frac{\partial^2 w(x,t)}{\partial x^2} - E\alpha\theta, \tag{8}$$

where $w(x,t)$ describes the deflection of the one-dimensional beam in the y direction at some position x.

Phonon hydrodynamics is an effective macroscopic method to investigate heat transport in nanostructures considering memory and nonlocal effects. Based on this method, the usual relation between the heat flux vector q_i and the temperature gradient considering nonlocal effects can be expressed as the form of Guyer–Krumhansl equation [18,19]:

$$q_i + \tau_0\dot{q}_i = -k\nabla T + l_2^2[\nabla^2 q_i + 2\nabla(\nabla\cdot q_i)]. \tag{9}$$

Eq. (9) generalizes the classical Fourier equation by adding to it relaxational effect, characterized by the relaxation time τ_0, and nonlocal effect, characterized by the phonon mean-free path l_2 (about 20–100 nm).

Substituting Eqs. (7) and (9) into Eq. (6) and replacing the operator ∇^2 with $\frac{\partial^2}{\partial y^2}$ [11], one can reach the heat transfer equation in