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# Lattice Boltzmann modelling of electro-thermo-convection in a planar layer of dielectric liquid subjected to unipolar injection and thermal gradient

Kang Luo [a,1], Jian Wu [b,1], Hong-Liang Yi [a,*], He-Ping Tan [a]

[a] School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, PR China
[b] GeoRessources Laboratory, Université de Lorraine (ENSG), CNRS, CREGU, F-54501 Vandoeuvre-les-Nancy, France

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ABSTRACT

In this paper, the electro-thermo-convective phenomena induced by the simultaneous action of a unipolar injection of ions and a thermal gradient in a dielectric liquid lying between two parallel plates are studied. We develop a lattice Boltzmann model (LBM) to solve the whole set of coupled governing equations, including the Navier–Stokes equations, the conservation equation of charge density, the Poisson's equation for electric potential and the energy equation. In this method, four different particle distribution functions are used to calculate the flow field, electric potential, charge density distribution and temperature field, respectively. A multi-scale analysis is also performed to recover the macroscopic equations from the discrete lattice Boltzmann equations (LBEs). The present method is validated with several carefully chosen test cases, and all LBM results are found to be highly consistent with available analytical solutions or other numerical works. Besides, the typical subcritical bifurcations and the hysteresis loops in electro-thermo-convective are clearly presented and analyzed.



## 1. Introduction

Electro-thermo-hydrodynamics (ETHD) is an interdisciplinary field dealing with the interactions between the free charges, electric field, flow motion and the thermal field [1]. The complex physics involved in electro-thermo-convective phenomena together with the promising applications in the active enhancement of heat transfer with electric field draw a wide attention to this field [2,3]. The complex physics essentially originates from the various coupling possibilities between different fields. For example, the flow field may be driven by the electric and thermal body forces, which in return contribute to the transport of free charges and heat. In addition, the thermal field may also affect the dynamic system through the temperature-dependent physical properties. In regard to heat transfer application, there are some attractive advantages for the techniques based on electric field, such as simple design, no moving mechanical parts, rapid response, smart control, low energy consumption and so on [4].

The complex mathematical model as well as the strong nonlinear couplings within ETHD problems has encouraged the application of direct numerical simulation approach to gain fundamental insights into many poorly understood electro-thermo-convective phenomena. Not surprisingly, most of previous numerical results with this topic are obtained with the conventional partial differential equation (PDE) based methods, such as the finite difference method (FDM) [5], the finite element method (FEM) [6], and the finite volume method (FVM) [7,8]. On the other hand, during the last two decades the lattice Boltzmann method (LBM) has experienced rapid development and has become a promising method for simulations of both simple and complex flows [9,10]. Only until recent years, LBM has been introduced into the electrohydrodynamic (EHD) field [11,12]. This is in contrast with the fact that LBM has long been applied in magnetohydrodynamics (MHD) since the early 1990s [13]. As a matter of fact, EHD and MHD can be viewed as two limiting cases of classical electrodynamics coupled to fluid mechanics [1].

Recently several lattice Boltzmann models have been proposed to analyze electrokinetic or electrohydrodynamic flows [14–18]. In most of these LBMs, a simplified model, i.e., the Poisson–Boltzmann (PB) model is adopted under the assumption of the thermodynamic equilibrium and ignores an ionic convection term [14,15]. This model leads to the partial decoupling of the electric field from the fluid flow, which greatly simplifies the solution procedure than with the fully coupled dynamic model. Later, Capuani et al. [12],

* Corresponding author.
*E-mail address:* yihongliang@hit.edu.cn (H.-L. Yi).
[1] These two authors contributed equally to this work.

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Wang et al. [16] and Yoshida et al. [17] proposed different hybrid or unified lattice Boltzmann models that simultaneously solve the Poisson's equation, the Nernst-Plank equations for concentration of different ion types, and the Navier–Stokes equations. The hybrid model here means that the mass and momentum conservation equations are solved by the LBM, while the electrostatics equations solved by the PDE based methods. By contrast, the LBM is applied to all governing equations with the unified model. Note that these models were originally designed for the electrokinetic flows in micro-/nano-scales, such as the electro-osmotic flow (EOF) in the micro-channel.

Quite recently, we proposed a unified lattice Boltzmann model for macro-scale electroconvective phenomena in isothermal dielectric liquids [18]. For the model problem of electro-convection induced by unipolar injection, our LBM accurately reproduced the subcritical bifurcation of the linear instability and finite amplitude bifurcation behaviours. The present study can be viewed as an extension of [18] to non-isothermal flows. We notice that the LBM has not been well established for ETHD problems. The main objective of this study is to present a unified and efficient LBM framework to solve the dynamic model for electro-thermo-hydrodynamic flows. As a starting point, we specially consider a planar layer of dielectric liquid lying between two parallel plate electrodes and subjected to a unipolar injection of ions and a thermal gradient. This is a standard model problem in ETHD, and it also serves as a good starting point for more sophisticated models. This problem has been extensively studied by means of the stability analysis approach [19], and the complete linear stability diagrams for different heating and injection configurations, various injection strengths, with and without residual conductivity have been obtained. In addition, several conventional PDE based methods have been developed for the same or similar mathematical model; see the review paper [20]. In [7] Traoré et al. numerically investigated the subcritical bifurcation behaviours of the configuration that both injection and heating are from the bottom electrode with a FVM solver. Their numerically obtained stability criteria show a good agreement with the values predicted by the stability analysis, which demonstrates the effectiveness and accuracy of their numerical method. Later their numerical method has been extended to consider other injection-heating configurations and temperature-dependent physical properties [8], and the more complex shapes and arrangements of electrodes [21]. The same FVM solver will be used in this study for some validation computations.

The remainder of this paper is organized as follows. In the next section, we describe the physical problem and the macroscopic governing equations and boundary conditions. In Section 3, we present the LBEs for the fluid velocity, electric potential, charge density and the temperature. The boundary condition treatment and the solution procedure are also explained in details in this section. In Section 4, the proposed LBM is validated, and numerical results are presented and discussed. The conclusions are drawn in the last section.

## 2. Physical models and macroscopic governing equations

As shown in Fig. 1, we consider a two-dimensional (2D) dielectric liquid layer of depth $H$ lying between two parallel and horizontally placed electrodes of length $L$. The two electrodes are maintained at fixed but different electric potentials and temperatures. The liquid is considered to be incompressible, Newtonian and perfectly insulating. We assume that the electro-chemical reaction at the liquid/electrode interface, which leads to the injection of ions, is the sole source for free charges in the bulk liquid. A detailed description of the injection process in either polar or non-polar liquids can be found in [1]. In order to simplify the discussion, the injection is further assumed to take place only at the bottom electrode (i.e., unipolar injection), and the injected charge density takes a constant value $q_0$ (i.e., homogenous and autonomous injection). Therefore, only one species of ions is involved in the system.

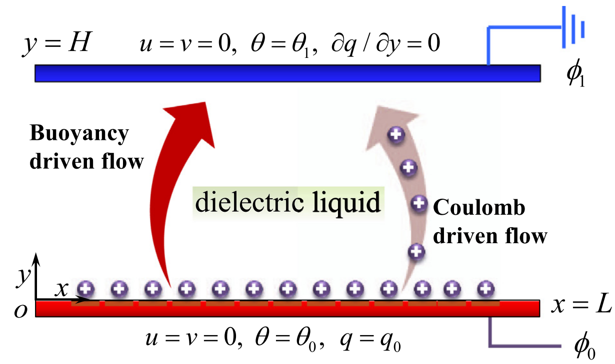


**Fig. 1.** Sketch of the physical domain: the dielectric liquid layer subjected to the buoyancy and Coulomb forces.

The set of equations governing the dynamic behaviour of the liquid subjected to the simultaneous actions of external electric and thermal fields at least include the electrical, mechanical and energy equations. For the physical problem under consideration in this study, under the Boussinesq approximation, the governing equations include the mass conservation equation of fluid (1), the Navier–Stokes equation (2), the Gauss's law (3), the definition equation of electric field (4), the charge conservation equation (5), and the energy equation (6), which may be expressed as [1],

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot (\rho_0 \mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial (\rho_0 \mathbf{u})}{\partial t} + \nabla \cdot (\rho_0 \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{u}) + \mathbf{f}_b, \tag{2}$$

$$\nabla \cdot (\varepsilon \mathbf{E}) = q, \tag{3}$$

$$\mathbf{E} = -\nabla \phi, \tag{4}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (qK\mathbf{E} - D\nabla q + q\mathbf{u}) = 0, \tag{5}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \nabla \cdot (\chi \nabla \theta), \tag{6}$$

where $\mathbf{u} = [u, v]$ is the fluid velocity, $\mathbf{E} = [E_x, E_y]$ the electric field and $\mathbf{f}_b$ the body force density. The scalars $\rho_0$, $p$, $\phi$, $q$ and $\theta$ denote the fluid density, hydrodynamic pressure, electric potential, charge density and temperature, respectively. The symbols $\mu$, $\varepsilon$, $K$, $D$, $\chi$, in turn, stand for the dynamic viscosity, electrical permittivity, ionic mobility, charge-diffusion coefficient and thermal diffusivity. In the above mathematical model, the magnetic effects and Joule heating have been ignored, since the electric current in insulating liquids are generally very weak. A quantitative justification for this simplification can be found in [22]. As shown in Eq. (5), there are three transport mechanisms for the charge density: (i) drift under the action of electric field, (ii) convection by the fluid velocity and (iii) charge diffusion. For the electro-convective and electro-thermo-convective flows considered here, the charge-diffusion coefficient takes a small value, which means the charge conservation equation is strongly convection-dominating with a modified convection velocity ($K\mathbf{E} + \mathbf{u}$).