Electric Power Systems Research 90 (2012) 117–125

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Endogenous computation of conjectured supply functions with network constraints

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ARTICLE INFO

Article history:
Received 17 June 2011
Received in revised form
29 December 2011
Accepted 12 March 2012
Available online 11 April 2012

Keywords:
Conjectured supply function equilibrium
Equilibrium with network constraints
Market splitting
Nash equilibrium

ABSTRACT

This paper presents a new iterative algorithm to compute a conjectured supply function electricity market equilibrium with DC transmission network constraints. This approach extends, to a network constrained system, a model previously developed by the authors for the single-bus case. At each iteration nodal prices are used to split the market into single prices areas. Since each area can be treated as a single-bus market from the transmission constraints' point of view, the single-bus algorithm is applied to compute the generators supply functions for each area. These new generators strategies are then cleared to determine new nodal prices and areas for the next iteration, and convergence is achieved when the network lines status and strategies of the generators do not change significantly in two consecutive iterations. The current approach has also been extended to deal with nodal elastic demands.

Unlike previous approaches, the main contribution of this work is that the parameters of the first order approximation of the conjectured supply functions (intercepts and slopes) are endogenously determined, coherently with the network lines status.

The algorithm has been applied to some illustrative case examples, and to a simplified version of the MIBEL market (Spain–Portugal). Results have shown to be very close to real data, and very relevant to analyze the economic impact of the capacity network constraints.

1. Introduction

Electricity markets regulate the interaction among generation companies and system demand to maintain the free competition's principles and the security of supply.

From the point of view of the generators, several approaches have been proposed to analyze their behavior for the medium and long term (see revisions in [1,2]). Most of these models formulate a simultaneous profit maximization of the generators, leading to an equilibrium point where no generator can increase its profit by a unilateral change in its strategy (Nash equilibrium [3]). These models differ in how they represent the generators' strategies, for example in terms of quantities (Cournot approach [4,5]) or supply functions [6,7]. Some of them estimate part of the parameters that define the generator's strategies from historical data (exogenous estimation) neglecting the influence of future changes in the regulation and/or the structure of the system, being more suitable for shorter forecasting horizons [1,8–10]. Others approaches include some additional constraints and/or reasonable hypothesis (e.g., linear supply function valid for all demand levels, see [6,11,12]) to formulate models that compute equilibrium strategies coherent with changes in both the regulation and the structure of the system (see [13] for a complete review).

Mathematical models can also differ in the degree of detail with which the system is modeled, or in the technical constraints considered [2]. These constraints can affect the generators' strategies leading to changes in the market results. Some of the constraints that play an essential role in medium and long term analysis are the network constraints and, in particular, the network capacity constraints [8,14,15]. A line gets congested when the power flow that results from the market equilibrium reaches its maximum capacity [14]. Congested lines may split a complete system into sub-systems or areas with different prices that behave as isolated markets (market splitting, see [16], and also [8] for an analysis of the effects of the network constraints). Each area can then be treated as a single-bus system from the congestions' point of view [17], naturally leading to different generators strategies for each area. From the point of view of the regulators, splitting into areas deteriorates competition (see [18]) since it may imply an increment of the market power of some generators.

There are few related works in the literature where the conjectural equilibrium with network capacity constraints is formulated.

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doi:10.1016/j.epsr.2012.03.008

These works can be classified into two groups: Cournot's conjecture approaches and general conjecture approaches.

The main characteristic of the Cournot's conjecture approach is that nodal generators' strategies do not consider the competitors' reaction, and all the generators located in the same area use the same conjecture which corresponds to the demand slope in that area, which must therefore be elastic and previously known [19,20]. The demand in each area is the aggregation of the nodal demands in that area.

On the other hand, the general conjecture approach generalizes the Cournot approach by including the competitors' reaction, allowing nodal inelastic demands. In this case the nodal generators' strategies are represented in terms of conjectured supply functions (CSF), which are linear approximations of the supply functions around the equilibrium point [1]. These functions usually depend partially on historical data, in the sense that if the intercepts are the decision variables, the slopes are fixed and estimated from past data, or vice versa. For example, Barquín et al. (see [10,17]) propose an iterative algorithm to solve a two-node conjectured approach when the intercepts are the decision variables and the network status is unknown. Assuming a network status to select the set of slopes to be used (slopes with or without congestions for each generator in each node), the market equilibrium is computed for each area by using an equivalent quadratic optimization problem (see [9,21]), and a new network status is obtained. The algorithm converges when the network status does not change in two consecutives iterations. Since only one line is considered, three iterations are enough to know if the algorithm converges when inter-temporal constraints are not taken into account. Similarly, Day et al. (see [1]) calculate the intercepts as an equilibrium result but considering the network status as known, using a linear mixed complementary problem (LMCP) with given slopes. Conversely, they also propose an approach to compute the slopes as an equilibrium result when the network status is known but the intercepts are assumed to be fixed. In this case the equilibrium is computed using a non-linear mixed complementary problem (NLMCP). Finally, Liu and Wu (see [8]) also propose an iterative algorithm to compute the slopes when the intercepts are fixed and equal to the intercepts of the linear marginal costs functions, and when congestions are unknown. In this case, taking into account the bids of the generators, the first stage solves a DC economical-optimal power flow to obtain the nodal productions and prices. These results are then used to calculate the generators' slopes solving a generators' maximization problem with fixed intercepts. The algorithm converges when, in two consecutive iterations, the slopes are equal.

This paper proposes a new fundamental approach to calculate the generators' strategies when DC network capacity constraints are considered. The proposed model, outlined in [22], generalizes a previous authors' approach developed in [23] for the single-bus case. Both approaches are CSF equilibrium, but do not assume supply function linearity along the overall prices range as in [6,11,12,24]. On the contrary, as the commented approaches [1,8,10,17], generators' supply functions are locally represented by their first-order approximations around the equilibrium. However, unlike [1,8–10,17,25,26], the proposed model computes both the intercepts and slopes of the first order approximations, assuming that they are robust against small inelastic demand changes. The main contribution of this paper is a market equilibrium model that computes the intercepts and slopes of the first-order approximation of the generators' supply functions consistently with the network status (in terms of congestions and single price areas). Fig. 1 shows a three axes classification of the existing approaches, and locates the proposed approach filling one of the existing gaps.

Refs. [8,10,17,19,20] show that the existence of the equilibrium with network constraints, when slopes or intercepts are fixed, cannot be guaranteed. However, since for the proposed algorithm both

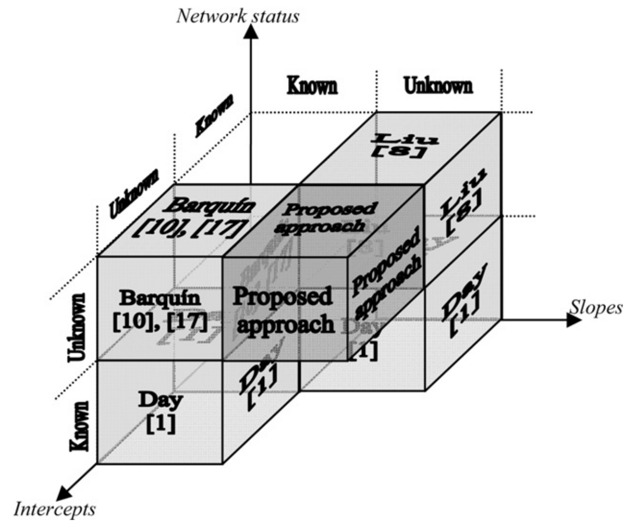

Fig. 1. Classification of CSF approaches with DC network constraints.

parameters are decision variables, these additional degrees of freedom relax the conditions for the existence of the equilibrium, as it occurs for example when extending a pure Nash equilibrium to mixed strategies (see [3]), and convergence was reached for all the tested cases.

The proposed methodology is based on an iterative two-step algorithm. In the first step, given a set of initial generators' supply functions, an optimal DC power flow is computed to obtain the nodal prices. Using these economic signals, nodes with same prices and interconnected by paths of nodes with same price (and thus with similar characteristic from the congestions' point of view) are grouped into single price areas (see [16,27–30]). Since each area can be considered as a single-bus system, the second step updates the generators' strategies by calculating the CSF equilibrium for each area as in [23]. Convergence is reached when the network status and generators' strategies do not change over two consecutive iterations.

The rest of the paper is organized as follows. Section 2 reminds the previous authors' single-bus algorithm and presents its novel extension to the elastic demand modeling. Section 3 extends the single bus algorithm to the case of network constraints. Section 4 shows its application to a simplified version of the Iberian market (MIBEL market that includes Spain and Portugal) and a meshed three-node synthetic case. Finally, Section 5 presents the conclusions drawn from this work.

2. Single-bus approach

2.1. Basic equilibrium equations

The electricity system can be modeled as a single-bus electricity market when network constraints are not considered [13,22,23]. The profit function B_i for each generator i is:

$$B_i(P_i, P_{-i}) = \lambda \cdot P_i - C_i(P_i) \quad (1)$$

where $C_i(P_i)$ is a non-decreasing and differentiable cost function for each generator, $\lambda = \lambda(P_i, P_{-i})$ is the system marginal price, P_i is the production of the ith generator and P_{-i} is the production of its competitors, that is:

$$P_{-i} = \sum_{j \neq i} P_j \quad (2)$$