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Topology optimization for the conduction cooling of a heat-generating volume with orthotropic material

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ABSTRACT

In this paper the two dimensional numerical topology optimization of a high conductive conduit material, distributed within a heat-generating material, is investigated with regards to the effect of orthotropic materials. Specifically, materials with orthotropic thermal conductivities (different primary and secondary principal thermal conductivities).

Two cases are considered in this study, namely the optimal distribution of an *isotropic conduit material* within an *orthotropic heat generating material*; and the optimal distribution of an *orthotropic conduit material* within an *isotropic heat-generating material*. A finite volume method (FVM) code, coupled with the method of moving asymptotes (MMA); the solid isotropic with material penalization (SIMP) scheme; and the discrete adjoint method, was used to find the optimal distribution of the high conductive conduit material within the heat generating material.

For the optimal distribution of an *isotropic conduit material* within an *orthotropic heat-generating material* is was found that a heat-generating material angle $10^\circ \leqslant \theta_0 \leqslant 60^\circ$ is preferred, for a higher thermal performance, and a heat-generating material angle $\theta_0 < 10^\circ$ and $\theta_0 > 60^\circ$ should be avoided.

For the optimal distribution of an *orthotropic conduit material* within an *isotropic heat-generating material* is was found that an optimal conduit material angle exists giving the best thermal performance (lowest $\tau_{\max}$). It was found that the optimal conduit material angle remains the same for different conductivity ratios and different heat-generating material angles. It was also found that the optimal conduit material angle directly corresponds to the domain aspect ratio, $\theta_{1,\text{opt}} = \tan^{-1}(2H/L)$, with a minimum improvement of 3% and a maximum improvement of 50% of the thermal performance when using an *orthotropic conduit material* over that of an *isotropic conduit material*. A 50% improvement of the thermal performance effectively translates to either double the allowable heat generation or half the peak operating temperature of the *isotropic heat-generating material*.

1. Introduction

The strive for higher power densities in electronics has been the driving force behind many of the miniaturization efforts, augmentations and unconventional ways of extracting heat from heat-generating volumes. Of interest, in many such investigations, is the optimization of conductive cooling paths (formed by a high conductive conduit material) within a heat-generating material with the condition that only a limited amount of volume be used as the conduit material. As described by Bejan [7], this volume-to-point or volume-to-surface type problem is one whereby heat is generated volumetrically (in a low conductive volume of given size) which must be channeled (through a high conductive conduit material) either to a point or surface on the boundary of the volume.

The volume-to-point or volume-to-surface type problem thus requires finding the optimal distribution of a limited amount of conduit material to augment the transport of heat within a heat-generating volume to predefined boundary regions. An optimal conduit material architecture leads to either a reduction in peak operating temperature at a given heat generation rate; or an increase in allowable heat generation rate and heat flux for a given peak operating temperature. This, in general, leads to the overall effect of higher effective power densities within the volume.

The augmentation of the volume-to-point or volume-to-surface type problem has been approached by a number of different methods, such as: utilizing constructal theory for various geometrically shaped volumes [1,7,8,27,4,13,31]; and embedding predefined

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Nomenclature

$\mathcal{A}$	coefficient matrix
$\mathcal{B}$	source term matrix
C	length of isothermal boundary
$f(\phi)$	objective function
$g_1(\phi)$	inequality constraint
H	domain height
k_r	thermal conductivity ratio
$\mathbf{k}$	thermal conductivity vector
k_1, k_2	conductivity components
k_x, k_y	conductivity components
L	domain length
N	increments for the penalization factor
$\dot{q}$	heat generation rate
$R(\Phi)$	residual
s, s_0	MMA asymptote factors
T	temperature
T_0	boundary temperature
T_{ave}	average temperature
T_{max}	maximum temperature
V	volume
V_t	total domain volume
x, y	Cartesian coordinates

Greek symbols

β	material orientation angle
θ	material angle
λ	adjoint vector
τ_{max}	non-dimensional thermal performance
ϕ	design variable
ϕ_{max}	volume ratio limit
Φ	arbitrary field
Ω	material domain

Subscripts

0	heat-generating material
1	conduit material
opt	optimum

Superscripts

I	optimization iteration
P	penalization factor

conduit material geometries within a heat-generating volume [14–16]. The optimization of predefined non-complex conduit material geometries are effective in augmenting the power densities with the added benefit of tailoring the conduit material geometry for ease of manufacturability. Such predefined geometries, however, impose restrictions on the conduit material architecture and may be far from the optimum solution as shown by Boichot et al. [10] and Song and Guo [32].

Approaches which involve shape and topology optimization have also been utilized to derive conduit material architectures. Li et al. [24,25] investigated two-dimensional heat conduction using an evolutionary structural optimization method. Gao et al. [18] investigated two-dimensional conduction problems using a modified bi-directional evolutionary structural optimization scheme. Boichot et al. [10] investigated cellular automaton with the goal of effectively cooling a heat-generating surface by arranging the configuration of high conductive material links. Cheng et al. [12] and Song and Guo [32] implemented bionic optimization in the construction of highly effective heat conduction paths. Xu et al. [35] investigated the volume-to-point problem using simulated annealing, which proved to perform better than constructal theory and bionic optimization.

Dirker and Meyer [17], Gersborg-Hansen et al. [19], Marck et al. [26] and Zhang and Liu [36] investigated the two dimensional topology optimization of the conduit material within a heat-generating volume for isotropic material. Burger et al. [11] extended this approach to a three dimensional study for a cubic domain with an isothermal boundary. It was found that the optimal topologies, obtained for the two- and three-dimensional studies, were similar to those of natural trees. Alexandersen et al. [2,3] also considered the topology optimization of two- and three-dimensional heat sinks cooled by natural convection, also leading to optimal topologies similar to natural trees.

In this paper, the method of moving asymptotes (MMA) [34] gradient based optimization algorithm is utilized, along with the solid isotropic with material penalization (SIMP) method and discrete adjoint method [33,20], in order to determine the optimal conduit material architectures. Specifically, for materials with orthotropic thermal conductivities (different primary and secondary principal thermal conductivities).

2. Numerical model

Fig. 1 shows the two dimensional computational domain and boundary conditions for the topology optimization problem considered in this paper. The internal volume (Ω_0 domain), of length L[m] and height H[m], generates heat at a rate of $\dot{q}_0$[W/m^3] and has an orthotropic thermal conductivity of $\mathbf{k_0} = (k_{x_0}, k_{y_0})$[W/mK]. A portion of the lower boundary (length C[m]) is isothermal with a temperature of T_0[K], while all other boundaries are adiabatic.

In order to augment the thermal performance of the system a conduit material (Ω_1 domain) is introduced with a high thermal

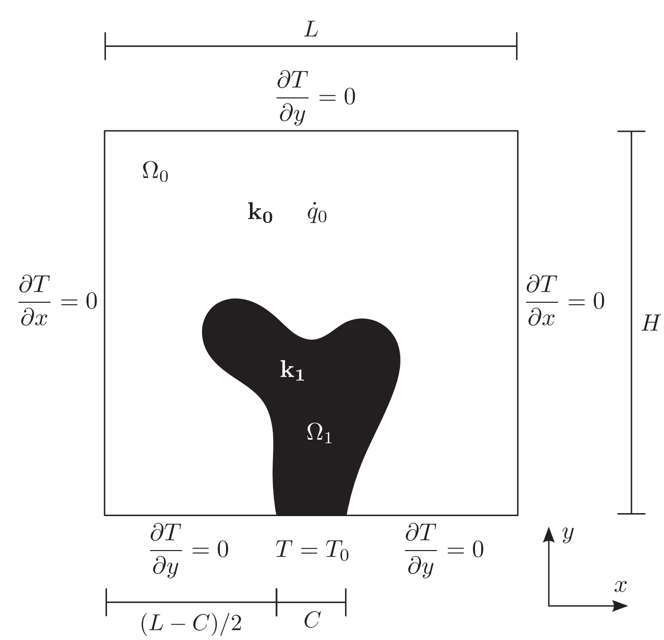

Fig. 1. 2D computational domain and boundary conditions for the topology optimization problem.