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Analysis of axial thermal conductivity of dual-porosity fractal porous media with random fractures

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ABSTRACT

Heat transfer in porous media has received steady attention in many applications such as in energy, aerospace, biomedicine and chemical engineering. In this work, based on the fractal characteristics of pores and fractures in porous media, the analytical solution of axial effective thermal conductivities is proposed for saturated dual-porosity media. It is found that the proposed axial effective thermal conductivity is a function of geometrical parameters of dual-porosity media, such as the porosities (ϕ_m, ϕ_f), fractal dimensions (D_f, D_l) for porous matrix and fractured network, tortousity fractal dimension (D_T), and fracture orientation (dip θ, azimuth α). The effects of the microstructural parameters on the effective thermal conductivity of the media are analyzed systematically. The model predictions are compared with the available experimental data, and good agreement between them is found. The present model may provide a better understanding of the physical mechanisms of heat transfer in dual-porosity media than conventional models.

1. Introduction

Heat transfer in porous media has steadily received much attention in science and engineering, for instance, heat extraction in hot dry rock, electronic cooling, proton exchange membrane fuel cell (PEMFC) and heat transfer in biological tissue, etc. [1–5]. The prediction of the thermal conductivity of porous media has been one of hot topics in the area of porous media for more than one hundred years.

In the past, many theoretical models for thermal conductivity of porous media were proposed [6–11]. Of them, parallel model and series model often provide predictions of upper and lower bounds of thermal conductivity for a two-component system. The mixed model is commonly used to estimate the thermal conductivity of new materials. Maxwell-Eucken model may be mostly used one, and Effective Medium Theory is an approximated approach to model the thermal conductivity of real media. Recently, a model was proposed by assuming small spheres dispersing into a medium based on the unit cell method [12]. The analytic solution of the effective thermal conductivity can unify the models mentioned above without weighting parameter. However, these theoretical models do not associate with the microstructure parameters such as pore distribution and tortuosity of micro-channel etc. Carson et al. [13] analyzed the advantages and disadvantages of each classic model and proposed the bounds of the effective thermal conductivity for isotropic porous media. But, the expression for the effective thermal conductivity was not given. With the development of computational technology, a large number of numerical methods such as finite element method [14], Monte Carlo method [15] and lattice Boltzmann method [16] were used to study the effective thermal conductivity of porous media. However, the results by numerical simulations were often expressed as empirical correlations, which often contain one or more empirical constants, and heat transfer mechanisms behind the empirical constants in empirical correlations were usually ignored. Besides, numerical simulation is generally time-consuming.

Over the past 20 years, fractal geometry theory was widely applied in random/disordered porous media for heat and mass transfer [17–20]. Furthermore, fractal geometry and technique were used to study heat and mass transfer in porous membranes. Xiao et al. [21] obtained a model for the permeabilities of water flow and gas diffusion in porous membranes by means of the fractal geometry and technique. Hao and Cheng [22] applied the fractal theory for porous media as well as the Lattice Boltzmann simulations to study the permeabilities in carbon paper gas diffusion layers. Yu and Cheng [23] analytically studied the effective thermal

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Nomenclature

k_e	effective thermal conductivity of a dual-porosity medium
$k_{e,m}$	effective thermal conductivity of a porous matrix
$k_{e,f}$	effective thermal conductivity of a fracture network
η	ratio of effective thermal conductivity of a fracture network to porous matrix
$R_{f,m}$	thermal resistance of fluid in pores
$R_{f,f}$	thermal resistance of fluid in fractures
R_s	thermal resistance of solid phase
ϕ_t	total porosity of a dual-porosity medium
D_f	fractal dimension for pore area
D_T	fractal dimension for tortuosity
ϕ_m	porosity of porous matrix
A_m	total cross-sectional area of a unit cell of porous matrix
τ	tortuosity
L_t	actual length of a tortuous capillary
L_0	straight length of a tortuous capillary
N	number of capillaries/pores
λ	pore diameter
$\lambda_{\max}$	the maximum pore diameter
$\lambda_{\min}$	the minimum pore diameter
s_m	cross-sectional area of a capillary
$q_{f,m}$	heat flow rate through a single capillary in porous matrix
$Q_{f,m}$	total heat flow rate through cross-section of a unit cell for porous matrix
ΔT	the temperature difference
D_l	fractal dimension for fracture area
ϕ_f	porosity of a fracture network
A_f	total cross-sectional area of a fracture network
θ	the mean dip angle of fractures
α	the mean azimuth of fractures
β	the proportionality coefficient depending on fracture scales
N_l	number of fractures
l	length for a fracture
$l_{\max}$	the largest length for a fracture
$l_{\min}$	the minimum length for a fracture
a	effective aperture of a fracture
s_f	cross-sectional area of a fracture
$q_{f,f}$	heat flow rate through a fracture
$Q_{f,f}$	total heat flow rate through cross-section of a unit cell for fracture networks

conductivity of bi-dispersed porous media based on the fractal geometry theory and technique. They found that the fractal model has the advantage over traditional models because the fractal model can be related to microstructure of a porous medium without empirical parameters. Kou et al. [24,25] analyzed the effective thermal conductivity of saturated/unsaturated fractal porous media based on the assumption that porous media consist of a bundle of tortuous capillaries. Watanabe and Takahashi [26] investigated the effect of micro-structural parameters of fractal fractured networks on heat extraction in hot dry rocks. In addition, Li et al. [27] applied the fractal-like tree networks to simulate the fracture networks to study the characteristics of heat transfer and fluid flow. Yu and Li [28] investigated the effective thermal conductivity of composite embedded with fractal-like tree networks by repeating a finite number of the elemental branches with H-shaped channels. Xu et al. [29] studied the effective thermal conductivity of Y-shaped fractal-like tree network. Miao et al. [30] derived the effective thermal conductivity of the damaged tree network and found that the damaged channel number and branching levels have significant effect on the optimal thermal conductivity. Recently, Li and Yu [31] obtained the expression for the effective thermal conductivity of a dual-porosity medium by assuming that the biological tissue consists of disordered fractal-like tree networks and tissue matrix. However, analytical expression for the effective thermal conductivity of *randomly fractured networks* was not reported. In addition, the above-mentioned fractal-like tree networks [31] were assumed to be a symmetric system, and matrix was assumed to be solid (without pores).

In reality, fractures in nature are random and disorder in spaces and lengths, and real fracture networks have been shown to have the statistically self-similar and fractal characteristic [32]. The dual-porosity media with porous matrix medium embedded with random fractures widely exists in nature, such as rocks and cracked soil [32]. In general, if a temperature gradient exists in a medium, in which a fluid is moving, the conduction and convection, respectively, occur across the medium between surface and fluid. If the porous medium has large pore size, the effect of convection is more apparent than heat conduction. However, convection can be ignored for small pore size (<4 mm) [33] on account of lack of intensive fluid-circulation in the pores. The heat flow rate across the parallel plate channels with free connection depends strongly on the geometrical ratio l/a [34]. They found that when $l \gg a$ and the Rayleigh numbers is small, heat convection may be neglected between two surfaces. In addition, there is thermal radiation in all surfaces of medium. However, Aduda [35] studied the effective thermal conductivity of porous media consisting of several loose particulates and found that thermal radiation in porous medium is ignored at low temperatures (<573 K). In this work, the porous medium with maximum pore size is smaller than 1 mm, the value of $a/l = 0.01$ is in the fracture networks based on requirements of the fracture morphology and the working temperature is lower than 573 K. Therefore, we focus on studying the characteristics of the heat transfer in a porous medium and fracture networks without thermal radiation and convection included. Although the process for heat transfer in dual-porosity media only has heat conduction, it is very difficult to obtain the effective thermal conductivity by traditional methods. This is due to the fact that both disordered pore distribution and random fracture morphology in space, which leads to the channels of heat transfer are extremely complicated. Furthermore, the analytical expression for the effective thermal conductivity of the dual-porosity media is rarely studied based on the fractal geometry theory and technique in literature. Therefore, in this work, we derive the analytical expression for the *axially* effective thermal conductivity of the dual-porosity media based on fractal distributions of pore/capillary sizes and fracture lengths. The validity of the proposed dual-porosity model will be verified by comparing the model predictions with the existing experimental data. Subsequently, the effects of micro-structural parameters of the dual-porosity media on effective thermal conductivity are systematically studied. The present model may provide the physical basis for better understanding of thermal transport in dual-porosity media.

2. Fractal characteristics of dual-porosity media

A dual-porosity medium usually consists of porous matrix medium and random fractures. Many researches showed that the distributions of pore sizes in porous matrix and fracture lengths have the fractal characteristics [23,32]. Generally, the scale of pore