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Non-linear radiative flow of three-dimensional Burgers nanofluid with new mass flux effect

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ABSTRACT

This article peruses the heat and mass transfer characteristics of three-dimensional steady flow of Burgers nanofluid over a bidirectional stretching surface. The convective boundary and nanoparticles mass flux conditions are considered. Additionally, the impact of non-linear thermal radiation and heat generation/absorption is delved. Further, the most recently proposed model for nanofluid is deliberated that necessitate nanoparticle volume fraction at the wall to be inertly rather than vigorously controlled. A set of similarity transformation is presented to alter the boundary layer equations into self-similar form and then tackled analytically by employing the homotopy analysis method (HAM). The effects of various controlling parameters to the heat and mass transfer characteristics are presented through graphs and scrutinized. The analytical out comes for the wall temperature gradient (Nusselt number) are calculated and presented through tables. It is seen that for enlarging values of the Brownian motion parameter lead to an attenuation in the concentration field as well as corresponding concentration boundary layer thickness. Likewise, it is noticed that the concentration field fall off hastily corresponding to Deborah number (β_3) in comparison to Brownian motion parameter. Moreover, in order to perceive the validity of the existing effort, the numerical outcomes are compared with the analytical solutions attained by the HAM and noted an outstanding agreement for the limiting cases.

1. Introduction

In spite of the fact that new evolutions in electronics enhance the performance of electronic devices. These advancements often mean the downsizing of these devices due to the power density concomitant with these components and it increases drastically which increases the heat flux spawned by the electronic components. The dissipation of this flux inside the device can lead to thermal problems such as overheating, which minimizes the devices enactment levels and their lifespan. As a result, it is a prodigious technological challenge to design and develop effective cooling systems. Such systems, will able to evacuate the substantial heat generated to maintain the temperature of electronic components below a certain prescribed value. New cooling methods are mandatory to be identified in order to encounter such challenges. In this milieu, the heat transfer rate of heat transfer devices can be elevated amongst other cooling technologies by adding additives to their working fluids. This alters the fluid transport properties and flow topographies. Choi [1] was the foremost figure who entitled the nanoparticles added to fluid nanofluid in 1995. Afterward, the exploration of this arena of new working fluid had been started. Furthermore, heat transfer utilizing nanofluids as coolant medium has emerged as one of the effective techniques in order to accomplish high heat dissipation. This possesses potential applications in precincts like compact heat exchangers, heat pipes etc. Keeping in mind, Oztop and Abu-Nada [2] scrutinized numerically the natural convection flow of nanofluids in partially heated rectangular enclosures. Boundary layer flow of viscous nanofluid towards a convectively heated plate was numerically addressed by Makinde and Aziz [3]. Pal and Mondal [4] scrutinized the MHD convective stagnation-point flow of nanofluids over a non-isothermal stretching sheet with induced magnetic field. Mixed convection stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with internal heat generation/absorption was studied by Pal and Mondal [5]. Heat and mass transfers effects in hydromagnetic flow of a viscous fluid with slip conditions and different types of nanoparticles were considered by Turkyilmazoglu [6]. Khan et al. [7] analyzed three-dimensional flow of an Oldroyd-B nanofluid towards a stretching sheet with heat generation/absorption. Their observations revealed that with the Brownian motion and thermophoresis parameters the

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temperature field increases however opposite behavior appeared for concentration field. Khan et al. [8] scrutinized the two-dimensional boundary layer flow and heat transfer to Sisko nanofluid over a stretching sheet in the concerned study. Their secured results showed that the temperature distribution increases with the thermophoresis and Brownian motion parameters. Kuznetsou and Nield [9] provided the revised model of natural convective boundary-layer flow of nanofluid past a vertical plate subject to the new proposed boundary condition. Hayat et al. [10] studied mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. Hayat et al. [11] developed the MHD 3D flow of nanofluid in presence of convective conditions. Hayat et al. [12] presented impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions. Khan and Khan [13] reported the forced convection analysis for generalized Burgers nanofluid flow over a stretching sheet. Khan and Khan [14] investigated the MHD boundary layer flow of a power-law nanofluid with new mass flux condition. Hayat et al. [15] studied the stretched flow of Carreau nanofluid with convective boundary condition. Recently, Hayat et al. [16] reported a model of solar radiation and Joule heating in magnetohydrodynamics (MHD) convective flow of thixotropic nanofluid.

An investigation on the radiative flow of forced convection problems has been incessantly attracting more attention. This is due to its intriguing industrial applications for instance, glass production, furnace design, nuclear power plants, comical flight aerodynamics rocket, propulsion systems, and space craft reentry aerodynamics that operate at high temperatures. Moreover, the intriguing effectiveness of thermal radiation is indispensable on the flow and heat transfer processes in the design of advanced energy conversion systems that maneuver at a high temperature. Besides, thermal radiation plays a substantial part in the heat transfer characteristic of absorbing-emitting fluids when convection heat transfer is small, predominantly in free convection problems. Likewise, the amount of thermal radiation manifestation within such systems is on account of emanation by hot walls and functioning fluids. It is well known that the blood flow regulates the temperature of the human body and controls it according to the environment. Nowadays, the thermal regulation in human blood flow by means of thermal radiation is very significant in several medical treatments for muscle spasm, myalgia, chronicwidespread pain and permanent shortening of muscle.

Over the past decade, studies of non-Newtonian fluids have greatly fascinated attention. The main reason being that many non-Newtonian fluids (such as humans blood, polymer suspension, slurries, and oil) are widely exist in life, production and nature. Amongst the non-Newtonian fluids, Burgers fluid model is of foremost importance due to its usefulness in describing rheological properties of many real fluids such as asphalt in geomechanics and cheese in food products. Moreover, Burgers fluid model is one of the most important fluid model which is used to model other geological structures such as Olivine rocks and the propagation of seismic waves in the interior of the earth. Several researchers investigated the Burgers fluid model including Jamil and Khan [17]. The steady flow of Burgers nanofluid over a stretching surface with heat generation/absorption was studied by Khan and Khan [18].

Motivated by the above-mentioned literatures and applications, the main emphasis of the present investigation is to explore the influence of surface heat source/sink on three-dimensional flow of Burgers nanofluid in the presence of non-linear thermal radiation. Similarity transformation is used to reduce the governing equations of the problem into a system of nonlinear ordinary differential equations. The obtained similarity equations are then solved analytically using the homotopy analysis method (HAM) [19–24]. The HAM does not require any small/large parameters in the problem. It gives us a way to verify the convergence of the developed series solutions. Moreover, it is useful in providing great freedom in the developing equation type of linear functions of solutions.

2. Formulation of the problem

We consider three-dimensional steady forced convective flow of an incompressible Burgers nanofluid over bidirectional stretching surface with linear velocities $u = ax$ and $v = by$, where a and b are taken as constants. Fluid flows over the region $z > 0$. The mass flux of the nanoparticles at the wall is assumed to be zero. Thermophoresis and Brownian motion effects are taken into account. Moreover, an assumption is made that the heated fluid under the sheet with temperature T_f is used to change the temperature of the sheet by convective heat transfer mode, which provides the heat transfer coefficient h_f. Here vicious dissipation is neglected. The equations governing the three-dimensional flow with heat and mass transfers can be expressed under usual boundary layer assumptions as (see Ref. [25]) (see Fig. 1)

$$
\begin{aligned}
& u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+w^2\frac{\partial^2 u}{\partial z^2}+2uv\frac{\partial^2 u}{\partial x\partial y}+2vw\frac{\partial^2 u}{\partial y\partial z}+2uw\frac{\partial^2 u}{\partial x\partial z}\right) \\
& +\lambda_2\begin{pmatrix} u^3\frac{\partial^3 u}{\partial x^3}+u^2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+2uv\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}+vu^2\frac{\partial^3 u}{\partial y\partial x^2}+2wu\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x^2}+wu^2\frac{\partial^3 u}{\partial z\partial x^2}-u^2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} \\
-u^2\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x^2}+2vu\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}+uv^2\frac{\partial^3 u}{\partial x\partial y^2}+2v^2\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}+v^3\frac{\partial^3 u}{\partial y^3}+2vw\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y^2} \\
-v^2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-v^2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}-v^2\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial y^2}+2wu\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial z^2}+uw^2\frac{\partial^3 u}{\partial x\partial z^2}+uw^2\frac{\partial^2 u}{\partial x^2} \\
+2vw^2\frac{\partial w}{\partial y}\frac{\partial^2 u}{\partial z^2}+vw^2\frac{\partial^3 u}{\partial z^2\partial y}+2w^2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}+w^3\frac{\partial^3 u}{\partial z^3}-w^2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2} \\
-w^2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2}-w^2\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial z^2}+uv\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y\partial x}+vu\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}
\end{pmatrix} \\
& +\lambda_2\begin{pmatrix} +u^2v\frac{\partial^3 u}{\partial y\partial x^2}+v^2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial x}+uv\frac{\partial v}{\partial y}\frac{\partial u}{\partial x\partial y}+uv^2\frac{\partial v}{\partial y}\frac{\partial^3 u}{\partial x\partial y^2}+wv\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial y}+uw\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y\partial x}+uvw\frac{\partial^3 u}{\partial x\partial y\partial z} \\
-uv\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y\partial x}-vu\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y\partial x}-uv\frac{\partial^2 w}{\partial y\partial x}+uw\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y\partial z}+vu\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial y\partial z}+uvw\frac{\partial^3 u}{\partial x\partial y\partial z} \\
+vw\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y\partial z}+v^2\frac{\partial w}{\partial y}\frac{\partial^2 u}{\partial y\partial z}+v^2w\frac{\partial^3 u}{\partial z\partial y^2}+w^2\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y\partial z}+vw\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial y\partial z}+vw^2\frac{\partial^3 u}{\partial y\partial z^2} \\
-vw\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y\partial z}-vw\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y\partial z}-vw\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial y\partial z}+uw\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial z}+u^2\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x\partial z}+u^2w\frac{\partial^3 u}{\partial x^2\partial z} \\
+vw\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial z}+uv\frac{\partial w}{\partial y}\frac{\partial^2 u}{\partial x\partial z}+uvw\frac{\partial^2 u}{\partial x\partial y\partial z}+w^2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+uw\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+uw^2\frac{\partial^2 u}{\partial x\partial z} \\
-uw\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x\partial z}-uw\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial x\partial z}-uw\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial x\partial z}
\end{pmatrix} \\
& =\nu\left[\frac{\partial^2 u}{\partial z^2}+\lambda_3\left(u^2\frac{\partial^2 v}{\partial x^2}+v\frac{\partial^3 u}{\partial y\partial z^2}+w\frac{\partial^3 u}{\partial z^3}-\frac{\partial u\partial^2 u}{\partial x\partial z^2}-\frac{\partial u\partial^2 v}{\partial y\partial z^2}-\frac{\partial u\partial^2 w}{\partial z\partial z^2}\right)\right], \qquad (1)
\end{aligned}
$$

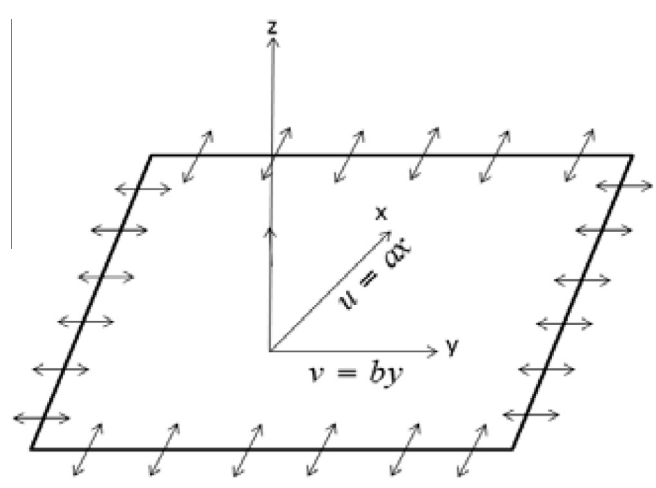

Fig. 1. Geometry of the problem.