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# Coupled equivalent circuit models for fluid flow and heat transfer in large connected microchannel networks – The case of oblique fin heat exchangers

Nasi Mou [a], Poh Seng Lee [a,*], Saif A. Khan [b]

[a] Department of Mechanical Engineering, Faculty of Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576, Singapore
[b] Department of Chemical and Biomolecular Engineering, Faculty of Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117575, Singapore

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ABSTRACT

This paper presents a simplified equivalent circuit model, which exploits the electric–hydraulic analogy and electric–thermal analogy, to predict the mass flow distribution and temperature distribution in an oblique fin array used in enhanced heat transfer applications. Methods to obtain accurate correlations for calculation of flow-dependent hydraulic 'resistances' are outlined and developed for both primary and secondary channels in the oblique fin array. Appropriate Nusselt number correlations and thermal resistance models are also employed to predict the temperature distribution associated with the mass flow distribution. Detailed full-domain numerical (CFD) simulations are performed to obtain parameters for the hydraulic resistance correlations, and also to serve as benchmarks for the proposed equivalent circuit model. Detailed comparisons between the results of simplified model and numerical simulation showed that the simplified model can accurately predict the mass flow distribution and temperature distribution, within ±5%, for varying fin number, aspect ratio, fin pitch, fin length, oblique angle and inlet velocity. Slightly higher deviations of mass flow prediction are observed for high inlet velocities as a result of the presence of vortices close to the trailing edge of the oblique fin region.


## 1. Introduction

Advancements in electronic packaging technology have led to smaller chip sizes, which in turn are associated with higher and more concentrated heat fluxes, which necessitate innovative and more effective cooling techniques, such as microchannel heat sinks. Numerous research studies into new fin designs and layouts designed to improve microchannel heat sink performance highlight secondary flows as one of the most promising heat transfer augmentation techniques [1–3]. Steinke and Kandlikar [4] proposed two potential methods in generating secondary flow for microchannel applications. The first is to add smaller channels at a certain angle between two main liquid channels, where the resultant pressure gradient drives the secondary flow from one main channel to the other. This design is already widely adopted in conventional flat tube-plate fin heat exchangers, where multiple louvered fins (acting as smaller secondary flow paths) are closely formed on the plate fin (main larger flow path). Alternatively, secondary flow can also be generated by a Venturi effect, by connecting a constriction area to the larger area section of an adjacent microchannel. Moreover, the use of twisted tape, helical ribs and screw threads has also proven effective to improve the convective heat transfer of laminar flow in tubes [5–7]. Another effective method to produce secondary flow is to mount or punch Vortex Generators (VGs) on convective heat transfer surfaces [8]. Offset strip-fin and pin–fins are also proven examples of flow disruption devices widely used in heat exchangers [9]. On the other hand, Tatsumi et al. [10] created notches on parallel plate fin arrays and these notches were either arranged parallel to each other or obliquely in the spanwise direction for comparison. Lee et al. [2] combined secondary flow generation and boundary layer reinitialization, and proposed the oblique fin design in microchannel heat sinks; by breaking the continuous fins into oblique sections, the heat transfer enhancement factor is close to 1.6 with Re ~ 300, compared to that of conventional microchannel heat sinks. Fan et al. [3] introduced the concept of oblique fins into the design of cylindrical heat sinks and obtained a much larger average Nusselt number (75.6% larger) compared with that of conventional straight fin heat sinks.

* Corresponding author. Fax: +65 6779 1459.
*E-mail addresses:* mounasi@u.nus.edu (N. Mou), pohseng@nus.edu.sg (P.S. Lee), saifkhan@nus.edu.sg (S.A. Khan).

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**Nomenclature**

| | |
|---|---|
| $A_{eq}$ | area of cross section, m$^2$ |
| $a$ | short side length of rectangular channel, m |
| $b$ | long side length of rectangular channel, m |
| $C$ | constants |
| $D_h$ | hydraulic diameter, m |
| $H$ | channel height, μm |
| $h$ | convective heat transfer coefficient, W/m$^2$·K |
| $I$ | electrical current, A |
| $k$ | thermal conductivity, W/m·K |
| $L$ | length, m |
| $Nu$ | Nusselt number |
| $P$ | perimeter, μm |
| $Po$ | Poiseuille number |
| $Q$ | mass flow rate, kg/s |
| $q$ | rage of heat flow, W |
| $R$ | resistance, Ω |
| $R_{hyd}$ | hydraulic resistance, Pa·s/kg |
| $R_t$ | thermal resistances, K/W |
| $Re$ | reynolds number |
| $V$ | electrical potential, V |
| $f$ | apparent friction factor |
| $l$ | fin length, μm |
| $l_u$ | fin pitch, μm |
| $m$ | number of fin per row |
| $n$ | number of fin row |
| $p$ | pressure, Pa |
| $u$ | velocity, m/s |
| $u_\infty$ | free stream velocity, m/s |
| $w_{ch}$ | main channel width, μm |
| $w_{ob}$ | oblique channel width, μm |
| $w_w$ | fin width, μm |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | aspect ratio |
| $\theta$ | oblique angle, ° |
| $\Delta$ | gradient |
| $\mu$ | dynamic viscosity, Pa·s |
| $\rho$ | mass density, kg/m$^3$ |
| $\nu$ | kinematic viscosity, m$^2$/s |

*Subscripts*

| | |
|---|---|
| $ch$ | channel |
| $m$ | mean |
| $main$ | main channel |
| $sec$ | secondary channel |
| $ob$ | oblique channel |

These advantages notwithstanding, the application of secondary flow to heat transfer enhancement may also have some drawbacks. As the coolant travels downstream, secondary flow generation and flow migration occurs continuously, thus leading to flow maldistribution in some configurations. Edge effects due to flow maldistribution may induce non-uniform temperature distributions along the heat sink footprint. When such a heat sink is used to cool an electrical component, the non-uniform temperature distribution might cause uneven thermal expansion of the device and could damage its electrical performance [11]. Various studies on louvered-fin arrays showed that bounding walls had a profound effect on fluid flow and heat transfer [12–15]. Springer and Thole [14] in their studies on flow field of louvered fins found that the flow field for a five row configuration indicated duct-directed flow while the flow field for both periodic and nineteen row arrays indicated louver-directed flow. Lee et al. [16] also found that flow migration occurred continuously as coolant traveled downstream due to secondary flow generation in planar oblique-finned microchannel arrays. Fan et al. [17] investigated the influences of the edge effect on flow and temperature uniformities for both partly blockaded and non-blockaded oblique-finned structure on cylindrical heat source surfaces through numerical and experimental studies. The flow field and temperature distribution analyses showed that the edge effect, which was present in the blockaded cylindrical oblique fin configuration, could cause the formation of localized hotspots due to the non-uniform flow distribution through the full domain minichannel heat sink. Thus, full domain simulations are needed when conducting numerical studies on planar oblique fin arrays to capture the edge effects.

However, full domain simulations are more cumbersome to set up and can be computationally expensive, which limit their use in practical engineering applications. As such, it is highly desirable to develop simplified models that are easy to use while at the same time reasonably accurate. Motivated by this, a simplified fluid flow model based on the electric–hydraulic analogy was proposed by Mou et al. [18] to complement full domain simulation on planar secondary flow generating configurations. The advantages of this new model are that it (1) greatly simplifies simulation procedures, (2) shortens the simulation time and (3) reduces the dependence on computer performance. This preliminary study showed that the method is feasible pending further development to translate the method into practical application and to improve its accuracy.

In this paper, a simplified equivalent circuit fluid flow model is developed to rigorously and accurately predict the flow distribution in planar secondary flow generating oblique fin configurations. The highly *nonlinear* relationship between pressure drop and mass flow rate is modeled via resistive electrical circuits with *nonlinear* current (flow rate)-dependent resistances. Detailed numerical simulations are employed to fit parameters in correlations for the hydraulic resistances of main and secondary channels, and also for subsequent model validation. A simplified equivalent circuit thermal model to predict temperature distribution in planar secondary flow generating oblique fin configurations is also proposed and validated in this paper. The findings of this study could be used to establish a fast and computationally efficient method to predict the performance of oblique fin arrays. More generally, this study serves as an example of representing complicated spatio-temporally varying heat and mass flows in the form of simpler, lumped circuit models, which can be used subsequently for design and optimization. Thus, our simplified nonlinear equivalent circuit model is generally applicable for studies on fluid flow and heat transfer performance of other heat sink arrays and can potentially facilitate the optimization of various configurations for heat transfer enhancement.

## 2. Theoretical approach

### 2.1. Electrical–hydraulic analogy

The analogy between electrical and mechanical (including fluidic) systems has been widely used since the early last century [19]. The well-known Hagen–Poiseuille law can be used to obtain an expression for the pressure drop $\Delta p$ in terms of the flow rate $Q$:

$$\Delta p = R_{hyd} Q \tag{1}$$