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Stability analysis of partitioned methods for predicting conjugate heat transfer

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ABSTRACT

The prediction of the heat transfer between a fluid and a solid object, known as conjugate heat transfer, is a very common problem in engineering sciences. This work investigates coupling methods which allow to solve such problems numerically by using separate solvers for both domains. The methods converge to the conjugate solution by exchanging boundary conditions at their interface. We review three known methods while postulating a forth novel method with improved stability properties.

Even though this coupling methods use standard solvers for each domain with known stability properties, many reports in the literature are found on instabilities occurring during the coupling procedure. While it is known that the origin of this problem lies at the exchange of boundary conditions, to date no closing stability criterion could be found.

The present work aims to provide a quantitative answer as to why these instabilities occur and to provide guidelines with respect to the use of the different methods. A new stability criterion is derived based on several simplifications. It shows that each method has its own stability limit and can be used within a specific range of applications, mainly dominated by the Biot number. Although the criterion is derived by making strong assumptions, it is validated through series of numerical experiments on a flat plate test case. It shows that we have correctly identified the mechanism leading to instabilities.

Finally, we compare the novel coupling strategy with the established methods. Considering the stability the new approach is advantageous especially for high Biot numbers, concluding that it can improve efficiency and accuracy of conjugate heat transfer computations.

1. Introduction

Conjugate heat transfer (CHT) is the transfer of heat between a solid and a surrounding fluid. It is prevalent in engineering sciences (e.g. [1,2]) and was first formulated by Perelman [3]. The correct prediction of conjugate heat transfer requires to simultaneously solve for the heat conduction in the solid and the Navier–Stokes equations in the fluid, where at the interface between both domains the temperature and heat flux have to be treated as unknowns and are found during the solution of the coupled problem.

Over the past half century, many analytical and numerical prediction methods have been proposed to deal with CHT problems. Two main strategies exist, depending on how the continuity of temperature and heat flux are imposed on the common wall between fluid and solid.

One approach integrates the entire set of equations in the fluid and solid as a single system and treats the continuity of temperature and heat flux implicitly. The coupled system of equations is solved together. This monolithic approach, in the literature sometimes referred to as the *conjugate method*, is computationally efficient, but requires that both the fluid and solid are put together into a unified framework.

A second approach calculates separately the flow and the thermal field with a coupling provided by the boundary conditions at the interface. This approach allows different stand-alone flow and solid platforms to be used within an iterative procedure to obtain the continuity of temperature and heat flux. The drawback of this partitioned approach, also known as the *coupled method*, is the need for sequential iterations between the two platforms which can lead to instabilities. Because in most cases different spatial discretizations are used in both domains and different grid refinements are imposed, an interpolation of the boundary conditions from one grid to the other is required. This, however, can also be present in a monolithic approach.

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Nomenclature

b	thickness of solid flat plate, [m]
Bi	Biot number
$B,\ K$	coefficients for analytical solution
c	speed of sound, [m s^{-1}]
h	heat transfer coefficient, [W m^{-2} K^{-1}]
L	characteristic length, [m]
q	heat flux, [W m^{-2}]
R	gas constant, [J^{1} K^{-1} mol^{-1}]
Re	Reynolds number
T	temperature, [K]
t	time, [s]
$u,\ v,\ w$	velocity components, [m s^{-1}]
α_0	error
β	relaxation coefficient
δ	boundary layer thickness
ν	kinematic viscosity, [m^2 s^{-1}]
μ	dynamic viscosity, [Pa s]
ρ	density, [kg m^{-3}]
λ	thermal fluid conductivity, [W m^{-1} K^{-1}]
CFD	computational fluid dynamics
CHT	conjugate heat transfer
FEM	Finite Element Method
FFTB	flux forward temperature backward
hFTB	heat transfer coefficient forward temperature backward
hFFB	heat transfer coefficient forward flux backward
RANS	Reynolds-averaged Navier–Stokes
TFFB	temperature forward flux backward
i	index for iteration
fl	fluid
s	solid
w	quantity at the wall
∞	free stream quantity

The fluid domain is governed by a set of non-linear partial differential equations, which are typically solved by an iterative technique. The solid domain, however, is governed by a linear elliptic partial differential equation which allows for a direct solution. This difference between both domains allows for different solution strategies of the coupled problem, where the exchange of boundary conditions can be put at different levels of advancement of the fluid solution, while the solid solver can be iterative as well as direct. To avoid confusion, we will reserve the term "iteration" from hereon exclusively for referring to a coupling iteration where both the fluid and solid simulations are run, while we will use "time step" to denote an advancement in the individual convergence of the fluid or solid domain, even though the iterative techniques used to solve both domains individually may differ from a time marching approach.

Even though CHT introduces only a weak coupling between two domains through the exchange of boundary conditions at their interface, the stability properties of the coupled system will in general be severely modified compared to the individual problems and a stability analysis is necessary.

Giles [4] provided a pioneering stability analysis of the solid–fluid coupling. The stability of a 1D model is analyzed by applying the stability theory of Godunov and Ryabenkii [5] on the discretized set of equations. Several simplifications are made, such as a uniform grid on both sides of the interface and the omission of the convection terms in the fluid domain. The latter one simplifies the fluid equations to the ones governing in the solid domain, however with a much lower conductivity. Giles concluded that the key point for achieving numerical stability is the use of Neumann boundary conditions (heat flux) for the structural calculation and Dirichlet boundary conditions (temperature) for the fluid calculations. However, this does not correspond to the stability behavior found in practice, as will be discussed in Section 3. The main reason is that Giles uses a time marching technique (both explicit and implicit) in solid and fluid domains, and updates the boundary conditions at every time step. A common practice for coupled methods is to use a steady state solver for the solid which provides immediately a steady state response to a given boundary condition and not a transient one as assumed by Giles. Similarly, in the fluid domain the boundary condition is only updated after a given number of time steps and not after each time step to decrease the computational cost.

Heselhaus [6] extended the theory of Giles by implementing convective boundary conditions for the solid domain and investigated the stability behavior if boundary conditions are exchanged after 2 time steps instead of one. It was shown that the method gains stability with increasing number of time steps.

More recently, Errera et al. [7] extended the theory of Giles further to coupling methods which use Robin boundary conditions at both domains, and for which the solid solver is a steady state one. They propose for the first time an optimal value for the heat transfer coefficient.

Most practical applications of the coupled method use a steady state solver for the solid domain while the boundary conditions for the iterative fluid solver are only updated after a sufficient number of time steps. This is motivated by a reduction in total computational time, where a balance is sought between converging the fluid domain on one side, and obtaining convergence of the fluid–solid coupling iterations on the other side. Too few time steps spent in the fluid domain would lead to a slow convergence of the coupling algorithm to settle the boundary condition, conversely, too many time steps would lead as well to a larger overall computational time, as time is lost on converging the fluid domain with wrong boundary conditions. A trade-off solution converges the fluid simultaneously with the boundary conditions and thus advances the fluid domain over multiple time steps before updating its boundary condition.

Most practical applications of the coupled method use a steady state solver for the solid domain while the boundary conditions for the fluid solver are only updated after a sufficient number of time steps. Indeed, an update at each fluid time step would only mean a small modification with respect to the previous state, while requiring a considerable computational cost since the steady state solution of the solid would be required. A large reduction in total computational time can thus be obtained if the boundary condition of the fluid is updated only after a sufficient level of advancement in time, which will lead to a significant change of the boundary condition for the next coupling iteration. This practice is, however, not reflected in the currently known methods for the stability analysis, and hence leads to a discrepancy between observed and predicted instabilities. The aim of the present work is thus to reconsider the stability of coupling schemes where boundary conditions are exchanged only after multiple time steps on the fluid and solid domains, to guarantee a fast convergence of the coupled problem.

The aim of the present work is thus to reconsider the stability of coupling schemes where only after a sufficient number of time steps on the fluid and solid domains exchanges of boundary conditions are performed.