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# Convective flow of carbon nanotubes between rotating stretchable disks with thermal radiation effects

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ABSTRACT

Flow and heat transfer effects of carbon nanotubes are analyzed between two rotating and stretchable disks are addressed. Two types of carbon nanotubes namely the single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs) with water as base fluid are used. Thermal radiation and convective boundary conditions are employed in modeling the heat transfer process. Appropriate transformations reduce the nonlinear partial differential systems to the ordinary differential systems. Convergent series solutions are obtained. Effects of various pertinent parameters on the velocity, temperature, skin friction coefficient and Nusselt number are shown and examined. It is found that water based single wall carbon nanotubes (SWCNTs) produce less drag and high heat transfer rate when compared with the water based multiple wall carbon nanotubes (MWCNTs).



## 1. Introduction

Convective heat transfer through nanoparticles is a popular area of research now a days. Nanofluid is an advanced kind of material containing suspension of solid particles called nanoparticles in traditional heat transfer fluids ($H_2O$, oil and ethylene glycol). Choi [1] firstly used the nanoparticles to enhance the thermal conductivity of fluids and storage of energy. It has gained much importance of the researchers due to enhanced thermal conductivity and convective heat transfer coefficient. Eastman et al. [2] described anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles. Tiwari and Das [3] discussed heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. Sheikholeslami et al. [4] analyzed effect of thermal radiation on magnetohydrodynamic (MHD) nanofluid flow and heat transfer by means of two phase model. Turkyilmazoglu [5] presented analytical solutions of single and multi-phase models for the condensation of nanofluid film flow. Zhang et al. [6] presented MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction. Shehzad et al. [7] examined thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field. Hayat et al. [8] analyzed MHD three-dimensional flow of nanofluid with velocity slip and nonlinear thermal radiation. Ellahi et al. [9] examined shape effects of nanosize particles in $Cu-H_2O$ nanofluid on entropy generation. Sheikholeslami and Ellahi [10] presented three dimensional mesoscopic simulation of magnetic field effect on natural convection of nanofluid. Sheikholeslami and Ellahi [11] also studied electrohydrodynamic nanofluid hydrothermal treatment in an enclosure with sinusoidal upper wall. MHD 3D flow of nanofluid in presence of convective conditions has been investegated by Hayat et al. [12].

Carbon nanotubes (CNTs) are cylindrical shapes like structures of carbon atoms with diameter ranges between 1 and 50 nm. These exhibit exceptional electrical, mechanical, thermal and optical properties at individual level. According to the details given by the CNT manufacturing firm NTI, at individual tube level CNT particles have; two hundred times strength and five times elasticity of steel, fifteen times thermal conductivity and thousand times the current capacity of copper and half the density of aluminum. Moreover CNTs don't possess any threat to the environment due to presences of carbon chains. Hence it is important to explore the effects of CNT on the fluid flow and heat transfer of Newtonian and non-Newtonian fluids. Model for thermal conductivity of carbon nanotube based composites has been presented by Xue [13]. Ding et al. [14] studied heat transfer of aqueous suspensions of carbon nanotubes. Kamali and Binesh [15] examined heat transfer enhancement using carbon nanotube when non-Newtonian nanofluids is used as a base fluid. Kumaresan et al. [16] explored convective heat transfer characteristics of secondary refrigerant based CNT nanofluids. Wang et al. [17] investigated heat transfer

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and pressure drop of nanofluids containing carbon nanotubes in laminar flows. Khan et al. [18] discussed fluid flow and heat transfer of carbon nanotubes along a flat plate with Navier slip boundary. Hayat et al. [19] examined homogeneous-heterogeneous reactions in the stagnation point flow of carbon nanotubes with Newtonian heating. Ellahi et al. [20] studied natural convection MHD nanofluid by means of single and multi-walled carbon nanotubes suspended in a salt water solution.

Fluid flow between rotating surfaces has gained much interest of researchers due to its engineering and industrial applications. The fluid flow due to rotating surfaces is used in electric-power generating system, air cleaning machine, medical equipment, gas turbines, food processing technology and aerodynamical engineering. Karman [21] initiated the work on flow by rotating disk. He transformed the Navier–Stokes equations to the ordinary differential equations. Later on various researchers have used the Von Karman transformation to discuss the different physical problems. Cochran [22] found more reliable solution using numerical integration. Disks rotating at different speeds are found in the internal cooling-air systems of most gas turbines. The rotor–stator system is used to model the flow and heat transfer associated with an air-cooled turbine disk and an adjacent stationary casing. The rotating cavity is used to model conditions between co-rotating turbine or compressor disks. Contra-rotating disks are used to model the wheel-space between the contra-rotating turbine discs of some existing (or future) engines. The flow between two rotating disks was first analyzed by Stewartson [23]. Lance and Rogers [24] examined axially symmetric flow of a viscous fluid between two infinite rotating disks. Afterwards the flow between a rotating and stationary disk was considered by Mellor et al. [25]. Heat transfer effects in the flow between two rotating disks have been studied by Arora and Stokes [26]. Kumar et al. [27] examined the flow of MHD fluid between a stationary impermeable disk and a rotating disk. Yan and Soong [28] studied the suction and blowing effects with transpiration of disks. Soong et al. [29] described the flow structure between two co-axial rotating disks. Jiji and Ganatos [30] obtained a numerical solution for the microscale flow and heat transfer between two rotating disks. In the recent years the investigation of flow over rotating stretchable surface has attracted the attention of research community due to its significant applications in different industries such as extrusion paper production, extrusion of polymers sheet, metal and plastic industries. Fang and Zhang [31] examined flow between two stretchable disks. Gorder et al. [32] obtained analytical solutions of a coupled nonlinear system arising in a flow between stretching disks. Hayat et al. [33] studied partial slip effect in flow of magnetite-$Fe_3O_4$ nanoparticles between rotating stretchable disks. Turkyilmazoglu [34] analyzed flow and heat simultaneously induced by two stretchable rotating disks. Hayat et al. [35] investigated MHD flow and heat transfer between coaxial rotating stretchable disks in a thermally stratified medium.

Sustainable energy generation at present is one of the serious issues the world over. Solar energy through minimal environmental impact thus offers a solution. Solar power is considered a natural way of obtaining heat, water and electricity from the nature. The radiation from solar energy and the resultant solar energized resources (wind, biomass, hydroelectricity, wave power etc.) give an explanation for most of the accessible renewable energy that is present on the earth. Solar energy is regarded as one of the best resource for clean and renewable energy. Makinde [36] examined second law analysis for variable viscosity hydromagnetic boundary layer flow with thermal radiation and Newtonian heating. Cortell [37] discussed fluid flow and radiative nonlinear heat transfer over a stretching sheet. Lin et al. [38] analyzed the effect of randomly-distributed droplets on thermal radiation of surfaces. Effect of magnetic dipole on viscous ferro-fluid flow past a stretching surface with thermal radiation has been investigated by Zeeshan et al. [39]. A model of solar radiation and Joule heating in magnetohydrodynamic (MHD) convective flow of thixotropic nanofluid has been presented by Hayat et al. [40]. Nanoparticles through scattering of the incident radiation allow higher levels of absorption within the fluid. Therefore the utilization of nanofluids in solar thermal system seems quite interesting area of research. Having such in mind, the purpose here is to focus on the effect of thermal radiation in the convective flow of carbon nanotubes between two stretchable rotating disks. The characteristics of water based single and multi-wall carbon nanotubes (SWCNTs and MWCNTs) between two rotating disks with convective boundary conditions are not explored up till now. Hence both nonlinear momentum and energy equations are computed by a modern technique namely the homotopy analysis method (HAM) [41–45]. Impacts of dimensionless parameters on velocity, temperature, skin friction coefficient and Nusselt number are examined graphically and in tabular form.

## 2. Problem formulation

Consider an axisymmetric flow of incompressible nanofluid confined between two parallel infinite disks. It is assumed that the lower disk is located at $z = 0$ while the upper disk is at a constant distance $h$ apart. Both the disks are rotating in axial direction with different angular velocities $\Omega_1$ and $\Omega_2$. Also both disks are stretching in radial direction with different stretching rates $a_1$ and $a_2$ (see Fig. 1).

Single and multi-wall carbon nanotubes are used as the nanoparticles with water as base fluid. Empirical correlations are used for the thermophysical properties of carbon nanotubes (CNTs) in terms of solid volume fraction. Further the surface exhibits convective type boundary conditions. We assume that the bottom surface of the disk is heated by convection from a hot fluid at temperature $T_0$ while temperature of upper disk is $T_1$. Characteristics of heat transfer are also explored with the help of radiation effects. Using cylindrical coordinate system $(r, \Theta, z)$, the governing equations for the considered flow are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + \nu_{nf}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right), \quad (2)$$

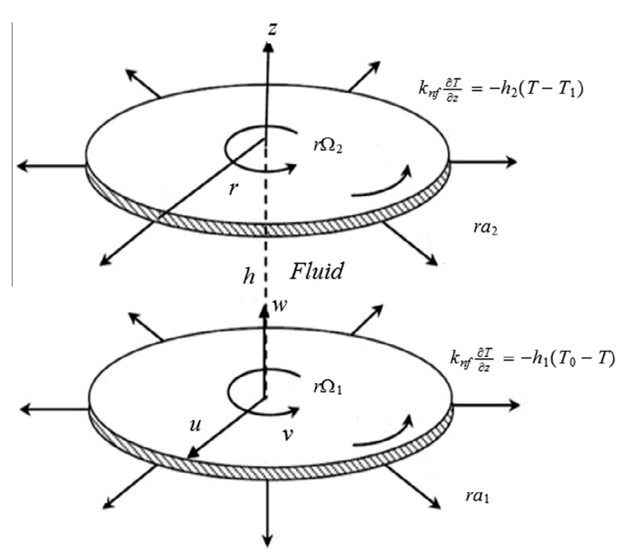


**Fig. 1.** Geometry of the problem.