International Journal of Heat and Mass Transfer 98 (2016) 401–414

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Review

Analytical determination of oscillating frequencies and onset temperatures of standing wave thermoacoustic heat engines

A.A. Boroujerdi, M. Ziabasharhagh [*]

Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran 19991-43344, Iran

ARTICLE INFO

Article history:
Received 22 November 2015
Received in revised form 6 March 2016
Accepted 8 March 2016

Keywords:
Standing wave
Thermoacoustic engine
Onset temperature
Oscillating frequency
Parallel plate stack
Binary gas mixture

ABSTRACT

This paper determines analytically the oscillating frequency and onset temperature of a simple standing wave thermoacoustic heat engine as eigenvalues of wave problem. The method to treat is based on the linear theory of thermoacoustics and the analytical solution of wave equation. The influence of geometrical parameters such as stack position, stack length, engine total length, and the influence of operational parameters such as charge pressure, on the oscillating frequency and onset temperature for four different working gases of Hydrogen, Helium, Nitrogen and Argon are investigated. Moreover, the eigenvalues are calculated for binary gas mixtures of four mentioned gases.

Contents

1. Introduction

Thermoacoustics was qualitatively described about 130 years ago by Rayleigh [1] with the following statements about air oscillations inside a tube: “at the phase of greatest condensation heat is received by the air, and at the phase of greatest rarefaction heat is given up from it, and thus there is a tendency to maintain the vibrations.” This was the basis of the twenty’s century researches which quantitatively studied thermoacoustic phenomena. The first main mathematical description of thermoacoustic effects was presented by Rott in 1969 [2]. He developed the mathematics describing the acoustic oscillations in a gas in a channel with an axial temperature gradient, with lateral channel dimensions of the order of the gas thermal penetration which are much shorter than the

---

[*] Corresponding author. Tel.: +98 21 8406 3242; fax: +98 21 8867 7274.
E-mail addresses: A.A.Boroujerdi@mail.kntu.ac.ir, A.A.Boroujerdi@gmail.com (A.A. Boroujerdi), mzia@kntu.ac.ir, masziab@gmail.com (M. Ziabasharhagh).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.03.028

Nomenclature

A	cross-sectional area
a	speed of sound
B	matrix of coefficients
C_P	isobaric specific heat
C_s	specific heat of solid
C_v	specific heat at constant volume
E	total energy flow
f_v	Rott's viscous function
f_k	Rott's thermal function
i	imaginary unit
Im	imaginary part
J_0	Bessel function of zeroth order
J_1	Bessel function of first order
k	thermal conductivity
L	length
l	half of plate thickness
L_S	length of stack
L_R	length of resonator
M	molecular mass
P	pressure
P_{ch}	charge pressure
Pr	Prandtl number
R	gas constant
Re	real part
r_0	radius of engine
T	temperature
t	time
U	volume flow rate
u	velocity
x_L	position of left end of stack
x_R	position of right end of stack
x	longitude coordinate
y_0	half of plate spacing

Greek letters

α	thermal diffusivity
γ	specific heat ratio
δ_k	thermal penetration depth
δ_v	viscous penetration depth
μ	dynamic viscosity
ξ	gas–solid capacity ratio
ρ	density
ϕ	porosity
ω	oscillating frequency
v	kinematic viscosity

Subscripts

0	mean value
1	acoustic value
S	solid

Superscripts

$\dot{}$ (dot)	time derivative
*	complex conjugate

wavelength. Subsequently, from 1973 to 1983, Rott presented several articles under various aspects of "Thermally Driven Acoustic Oscillations". Since 1983, Swift [3,4] has extensively promoted the quantitative and qualitative description of thermoacoustic phenomena and utilities based on the Rott's researches. Afterwards, many researchers have studied thermoacoustic systems (standing wave, traveling wave, and cascade).

Onset of the self-excitation of a thermoacoustic device is of the most interesting phenomena to be investigated along with its two key characteristics: onset temperature of hot source and first mode natural frequency. Like all the heat engines, a thermoacoustic engine works with the temperature difference between the hot source and the cold sink at the two sides of the stack. Initiating the oscillations in the thermoacoustic engine requires a critical temperature gradient along the stack. Fixing the cold end of the stack at ambient temperature, there is a lower limit of the hot-end temperature of the stack, at which self-excited oscillations occur and growth toward a stable condition with natural frequency. This temperature and corresponding heat input are usually called the onset temperature and the onset heat input, respectively. The stability limit is expressed in an amount of heat, or a temperature difference in the engine, for which neutrally stable oscillations can exist. Summarily, oscillation frequency and onset temperature are two basic characteristics of thermoacoustic oscillations in relevant engines for which three main categories of experimental, analytical and numerical procedure are implemented.

Frequency of oscillations can be achieved experimentally, numerically, and analytically. To calculate the frequency of oscillations, analytically, Tu et al. [5] proposed a network method based on the principle of the impedance matching at components interfaces. The method is straightforward, but leads to a complex frequency value which has no solid physical background. Later, Dai et al. [6] introduced a simple method for frequency calculation through numeric investigation. According to their iterative method, frequency can be decided if it leads to an inflection point of the amplitude of volume flow rate, which is also a local minimum, most close to the volume flow rate node boundary. Compared with experimental data, the method proved to be very reliable.

Onset temperature is of great importance besides the frequency because the hot end temperature of stack must be controlled below the overheating temperature of solid materials (mostly about 900 K) due to safety requirements. Experimental works were done on the onset and damping processes of the oscillation in standing wave [7], traveling wave [8], and cascade [9] thermoacoustic engines. The effects of different working gases on the performance and onset characteristics of a solar powered standing wave thermoacoustic engine and a small traveling wave thermoacoustic engine were investigated experimentally by Shen et al. [10] and by Chen et al. [11], respectively. Hariharan et al. experimentally studied the influence of operational and geometrical parameters on the performance of a twin thermoacoustic heat engine and compared the results to the results attained by DeltaEC [12–14]. The impact of regenerator hydraulic radius, resonator length, and the mean pressure on the characteristics of a looped traveling wave thermoacoustic heat engine is investigated by Yu et al. [15]. The experimental results indicate that the ratio of penetration depth over hydraulic radius plays an important role in the excitation and pressure amplitude of the two acoustic modes and there is a measured optimal relative penetration depth, which leads to the lowest onset temperature difference.

A.T.A.M. de Waele [16] treated the basic onset and damping features of the traveling-wave thermoacoustic engines. By the assumption of the constant hot-end temperature, A.T.A.M. de Waele described the dynamics of the system by a fourth-order