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Stability analysis of convective flows in a spherical gap under the influence of axial and dipolar magnetic fields

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ABSTRACT

This paper describes a numerical investigation of the influence of axial and dipolar magnetic fields on the stability of the convective flow of an electrically conductive Boussinesq fluid in a spherical gap for the radius ratios η = inner radius/outer radius = 0.4, 0.5, and 0.6. The inner shell is warmer than the outer shell. We show that whereas the axial magnetic field stabilizes the flow for low Hartmann numbers, a destabilizing effect is detected if the magnetic field increases. On the other hand, numerical analysis shows that the dipolar magnetic field configuration stabilizes the flow. The critical wave number, m_c, is much higher than that for the axial magnetic field. The critical Grashof numbers are presented as a function of the Hartmann number for both configurations of the B-field. The stability analysis is accompanied by calculations of the 3D flows.

1. Introduction

The investigation of convective flows in spherical geometries plays an important role in many technical and industrial applications, e.g. solidification [1,2], and energy storage. Furthermore, buoyancy-driven flows are of interest in fundamental processes such as the investigation of nonlinear effects, leading to a rich variety of bifurcations and transitions to chaos [3–5].

Let us suppose that a steady external magnetic field is applied. Magnetohydrodynamic effects become important if the electrical conductivity of the working fluid is high. Usually, liquid metals such as mercury or In-Ga-Sn ($\approx 10^6 \Omega^{-1}\,\mathrm{m}^{-1}$) are used. Note that because of low kinematic viscosity and high thermal diffusivity, the Prandtl number is low ($Pr = 0.015$ is used in this study).

The next issue is the kind of B-field to be applied. When restricted to steady fields, there are few possibilities. First, the magnetic field should be axisymmetric. This choice of B-field does not change the axisymmetric structure of the basic flow. Therefore, after the evaluation of the basic flow, linear stability analysis can be performed, and the influence of the B-field on the stability of the basic flow can be determined. Second, the magnetic field should obey a condition $\nabla \times \mathbf{B} = 0$. If this condition is not satisfied, additional electric currents which influence the flow occur. Taking into account these two conditions, two B-field configurations are most natural: axial $B_{axial} = B_0(\cos\theta \mathbf{e}_r - \sin\theta \mathbf{e}_\theta)$ and dipolar $B_{dipolar} = B_0(2r^{-3}\cos\theta \mathbf{e}_r + r^{-3}\sin\theta \mathbf{e}_\theta)$. We know from the literature dealing with the influence of axial [6–8] and dipolar [9] fields and combinations of both [10] on the spherical Couette flow (SCF) that the fields produce different flow structures, i.e. due to the formation of unexpected boundary-layer-stability regimes and bifurcation scenarios. Therefore, it is important to investigate the influence of the above fields on convective flows.

This study is, on the one hand, a continuation of [11], in which the convectively driven flow was investigated under the influence of an *axial* magnetic field. The stability problem is solved in [11] for Hartmann numbers from 0 to 20, where the radius ratio η changes from 0.4 and 0.8. We found that the axial magnetic field stabilizes the convective flow, i.e. the critical Grashof number, Gr_c, increases with an increase in Ha. We expanded upon this research, considering the same problem for larger Hartmann numbers, and found that the curve of the marginal stability had an unusual shape. If the Hartmann number exceeds a particular value, the critical Grashof number drastically decreases. Therefore, an external axial magnetic field destabilizes buoyancy-driven flows.

On the other hand, new research discussing the influence of the *dipolar* magnetic field on the convective flow has been presented. It has been established that the dipolar magnetic field makes the basic flow very stable. This stabilization effect is accompanied by a drastic increase in the critical azimuthal modes, m_c.

An additional motivation for these investigations is the similar analysis of the influence of the *axial* magnetic field on the stability of the convective flows in rectangular [12,13] and cylindrical [14]

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cavities. Whereas with the rectangular geometry the stability curves, i.e. $Gr_c(Ha)$, have been established as displaying complex shapes, in the cylindrical cavity, the axial magnetic field is seen to have a strongly stabilizing effect on the convective flow. Therefore, performing the stability analysis for some radius ratios η in the spherical geometry improves our understanding of the influence of magnetic fields on convective processes.

The direct numerical simulations of the three-dimensional flows performed for both configurations of the B-field not only confirm the results obtained via the linear stability method but also provide better understanding of the mechanism of the instability.

This paper is organized as follows. After the problem is formulated in Section 2, the numerical method is presented, accompanied by test calculations (Section 3). The structure of the basic flow for both structures of applied magnetic field is discussed in Section 4. Methods for investigating the stability are formulated in Section 5. Section 6 records the results of the stability analysis in terms of linear stability theory. An investigation of stability is accompanied by an energy analysis using the Reynolds–Orr equation. Nonlinear calculations of the supercritical flows are presented in Section 7.

2. Equations

A Newtonian fluid with a kinematic viscosity ν, a density ρ, and an electrical conductivity σ between two spherical surfaces with an inner radius R_1 and an outer radius R_2 is considered. The inner surface is maintained at a constant temperature T_1; the outer surface is maintained at T_2 such that $T_1 > T_2$. The Boussinesq approach is used to describe the buoyancy force: $\rho = \rho_0(1-\beta(T-T_2))$ (see Table 1). The system is subjected to either an *axial* magnetic field $\mathbf{B}_a = B_0(\cos\theta\mathbf{e}_r - \sin\theta\mathbf{e}_\theta)$ or a *dipolar* magnetic field $\mathbf{B}_d = B_0(2\cos\theta\mathbf{e}_r + \sin\theta\mathbf{e}_\theta)/r^3$ of constant magnitude B_0. The magnetic Reynolds number is $Rm \ll 1$; therefore, the low Rm approximation can be used [15].

The non-dimensionalized equations are

$$\frac{\partial\mathbf{u}}{\partial t} - \mathbf{u}\times\nabla\times\mathbf{u} = -\nabla p + GrT\mathbf{e}_z + \Delta\mathbf{u} + \mathbf{F}_L, \quad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \frac{1}{Pr}\Delta T, \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0. \quad (3)$$

The length has been scaled by the gap width D. The time has been scaled by the viscous timescale D^2/ν. The velocity field has been scaled by ν/D. The pressure has been scaled by $\rho\nu^2/D^2$. The temperature has been scaled by $\Delta T = T_1 - T_2$. The magnetic field has been scaled by B_0. Here, $Gr = \beta g\Delta TD^3/\nu^2$ is the Grashof number and $Ha = B_0D\sqrt{\sigma/\rho\nu}$ is the Hartmann number. The Lorentz force $\mathbf{F}_L$ takes the form $\mathbf{F}_L = Ha^2\mathbf{J}\times\mathbf{e}_{a,d}$. The electrical current $\mathbf{J}$ can be found using Ohm's law and conservation of charge

$$\mathbf{J} = (-\nabla V + \mathbf{u}\times\mathbf{e}_{a,d}), \quad (4)$$

$$\nabla\cdot\mathbf{J} = 0. \quad (5)$$

A Poisson equation for the electric potential V

$$\Delta V = \mathbf{e}_{a,d}\cdot\nabla\times\mathbf{u} \quad (6)$$

must be solved together with Eqs. (1)–(3). The boundary conditions for the temperature are:

$$T_{\frac{\eta}{1-\eta}} = 1, \quad T_{\frac{1}{1-\eta}} = 0. \quad (7)$$

The no-slip conditions for the velocity field are given by

Table 1
List of parameters.

$\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi$	unit vectors in radial, polar and azimuthal directions
$\mathbf{e}_a$	$\cos\theta\mathbf{e}_r - \sin\theta\mathbf{e}_\theta$
$\mathbf{e}_d$	$2r^{-3}\cos\theta\mathbf{e}_r + r^{-3}\sin\theta\mathbf{e}_\theta$
r	radial coordinate
x	radial coordinate ($x\in[-1,1]$), $r(x) = \frac{1}{2}\left[x + \frac{1+\eta}{1-\eta}\right]$
$\mathbf{u}$	velocity field
u_r, u_θ, u_ϕ	radial, polar and azimuthal velocity components
$\mathbf{u}_0$	velocity field (basic flow)
$u_{0r}, u_{0\theta}$	radial and polar velocity components (basic flow)
$\tilde{\mathbf{u}}$	velocity field perturbation
t	time
Δt	time step
p	pressure
$\tilde{p}$	pressure perturbation
B_0	applied magnetic field
V,T	electric potential, temperature
V_0, T_0	electric potential, temperature (basic flow)
$\tilde{v}$	electric potential perturbation
$\mathbf{F}_L$	Lorentz force
$\tilde{\mathbf{F}}_L$	Lorentz force perturbation
$\mathbf{J}$	electrical current density
m_c	critical wave number
Gr	Grashof number, $\beta g\Delta TD^3/\nu^2$
Ha	Hartmann number, $B_0D\sqrt{\sigma/\rho\nu}$
Pm	magnetic Prandtl number, ν/α
Pr	Prandtl number, ν/κ
Rm	magnetic Reynolds number, $Rm = UD/\alpha$
R_1, R_2, D	inner radius, outer radius, $R_2 - R_1$
Greek symbols	
α	magnetic diffusivity
β	volume expansion constant
ν	kinematic viscosity
η	radius ratio, R_1/R_2
κ	thermal diffusivity
σ	electrical conductivity
ρ	density
λ	eigenvalue ($=\gamma + i\omega$)
ω	frequency of critical mode and $Im(\lambda)$
Φ_0	poloidal function of the basic flow
$\tilde{\Phi}, \tilde{\Psi}$	poloidal and toroidal functions of the perturbation
Θ	perturbation for temperature
Φ, Ψ	poloidal and toroidal functions
χ	stream function of the basic flow

$$u_r = 0, \quad u_\theta = 0, \quad u_\phi = 0 \quad (8)$$

on $r = \eta/(1-\eta)$ and $r = 1/(1-\eta)$.

Furthermore, for electrically insulated surfaces, the following boundary condition

$$\frac{\partial V}{\partial r} = 0 \quad (9)$$

holds on both surfaces.

3. Numerical method and test calculations

An equation system in Eqs. (1)–(6) with the boundary conditions in Eqs. (7)–(9) has been solved using the pseudospectral method using the numerical code developed by R. Hollerbach [16]. The velocity field can be represented in terms of the poloidal (Φ)-toroidal (Ψ) potentials as follows:

$$\mathbf{u} = \nabla\times\nabla\times(\Phi\mathbf{e}_r) + \nabla\times(\Psi\mathbf{e}_r). \quad (10)$$

Substituting Eq. 10 into Eq. (1) and applying the operations *rotrot* and *rot* not only eliminates pressure but also leads to *separated* equations for the potentials Φ and Ψ. Therefore, instead of one vector equation, as in Eq. (1), we obtain two scalar equations: a fourth-order equation for Φ and a second-order equation for Ψ: