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An interface integral equation method for solving transient heat conduction in multi-medium materials with variable thermal properties

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ABSTRACT

In this paper, a new single interface integral equation method is presented for solving transient heat conduction problems consisting of multi-medium materials with variable thermal properties. Firstly, adopting the fundamental solution for the Laplace equation, the boundary-domain integral equation for transient heat conduction in single medium is established. Then from the established integral equation, a new single interface integral equation is derived for transient heat conduction in general multi-medium functionally graded materials, by making use of the variation feature of the material properties. The derived formulation, which makes up for the lack of boundary integral equation in solving multi-medium problems, has the feature that only a single boundary integral equation is used to solve multi-medium transient heat conduction problems. Compared with conventional multi-domain boundary element method, the newly proposed method is more efficient in computational time, data preparing, and program coding. Based on the implicit backward differentiation scheme, an unconditionally stable and non-oscillatory time marching solution scheme is developed for solving the time-dependent system of differential equations. Numerical examples are given to verify the correctness of the presented method.

1. Introduction

With the advantages of semi-analytical feature and dimensional reduction characteristic, the boundary element method (BEM) has been successfully applied to solve transient heat conduction problems [1–4]. According to the differences of solution procedures, most of the existing approaches can be classified into two broad categories: the transformed space approach (Rizzo and Shippy [5]; Sutradhar et al. [6]; Sutradhar and Paulino [7]; Simoes [8]; Guo et al. [9]), and the time domain approach (Wrobel and Brebbia [10]; Ochiai et al. [11]; Tanaka et al. [12]; Yang and Gao [13]; Al-Jawary et al. [14]; Yu et al. [15]). In the transformed space approach, the time dependent derivative is removed by applying an algebraic transform variable, and the system of equations is solved in the transform space, then inverse transform is employed to reconstitute the solution in time domain. This method does not require time marching, and usually leads to an accurate result. However, it is very difficult to determine the transformation parameter, and for many practical problems, a large number of sampling frequencies is required to obtain an accurate solution, therefore the numerical inverse transformation is very time-consuming and an accelerated technique is usually needed [6]. The other kind is the time domain approach, by which the solutions are found directly in the time domain. One implementation of the time domain approach is the use of time-dependent fundamental solution [10,11], that can result in a pure boundary integral equation algorithm. However, numerically evaluating the boundary integrals requires both space and time discretization. More details about time-dependent fundamental solution approaches can be found in the works of Wrobel and Brebbia [10] and Ochiai and Sladek [11]. Another implementation of the time domain approach is to employ the fundamental solution for the Laplace equation, and transform the volume integrals associated with time dependent derivative into equivalent boundary integrals. Among the transforming techniques, the dual reciprocity method (DRM) [16,17], Multiple reciprocity method (MRM) [18], and radial integration method (RIM) [19] are most widely used.

For non-homogeneous materials, Kassab and Divo [20,21] derived a generalized fundamental solution for steady heat conduction in isotropic and anisotropic media with spatially varying thermal conductivity. Then by adopting the generalized fundamental solutions for non-homogeneous media, a generalized dual reciprocity BEM is presented for solving transient heat conduction problems in non-homogeneous media [22]. Based on a moving

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least square approximation of physical fields, Sladek and Sladek [23,24] proposed a local boundary element method for transient heat conduction analysis in functionally graded materials.

Taking the advantages of both finite volume and finite element methods, the control volume-based finite element method (CVFEM) can be used to simulate multi-physics problems in complex geometries [42–45]. By using CVFEM, Soleimani et al. [42,43] studied influence of an external magnetic field on ferrofluid flow and heat transfer in a semi annulus enclosure with sinusoidal hot wall [42], and investigated the force convection heat transfer in a lid driven semi annulus enclosure [43]. By using Lattice Boltzmann Method (LBM), Soleimani el al. [46] investigated the magnetohydrodynamic free convection flow of CuO – water nanofluid in a square enclosure with a rectangular heated body.

Transient heat conduction BEM has been broadened to a wide range of engineering problems, including non-homogeneous, anisotropic, and non-linear problems. But most studies mainly focus on single medium. However, most engineering problems involve objects composed of different materials. Therefore, it is important to develop a multi-medium BEM to solve wider range of engineering problems. The conventional widely used technique solving multi-medium problems is the multi-domain boundary element method (MDBEM) [25–29]. The basic idea of this method is that the whole domain of concern is broken up into a number of separate sub-domains, then a boundary integral equation is written for each sub-domain, and the final system of equations is formed by assembling all contributions of the discretized integral equations for each sub-domain based on the compatibility condition and equilibrium relationship. Adopting the combination of domain decomposition technique with a region-by-region integration algorithm, Divo and Kassab [30] developed a parallel BEM algorithm for solving large-scale, nonlinear heat conduction problems. In the transient heat conduction field, Erhart et al. [31] developed a parallel domain decomposition Laplace transform BEM algorithm for solving the large-scale transient heat conduction problems. Recently, Gao et al. [25,32,33] proposed a three-step multi-domain BEM for solving multi-medium non-homogeneous problems.

Although MDBEM is flexible in solving multi-medium problems, it has disadvantages in data preparation and computational time, since twice the element information over the same interface needs to be defined for the adjacent two sub-domains, and twice integrations need to be carried out over interface elements. Moreover, the variable condensation and assembling processes require a higher coding skill to develop a universal program, which heavily influences the computational efficiency. Tracing the issue to its source, the existing boundary integral equations were established on a single medium assumption, therefore it is awkward to solve multi-medium problems through using MDBEM, which involves tedious domain decomposing and assembling processes.

Recently, Gao and his coworkers proposed a single integral equation method, named interface integral BEM (IIBEM), for solving multi-medium problems [34–37]. Through a degeneration method from domain to interface integrals, the integral equation for solving single medium problems can be extended to interface integral equation capable of solving multi-medium steady heat conduction [34], elasticity [35,36] and elastoplasticity [37] problems. Comparing with the conventional boundary integral equation, an additional interface integral appears in the basic integral equation, embodying the difference of material properties between two adjacent media. The derived formulations made up for the lack of a boundary integral equation in solving multi-medium problems. Compared with MDBEM, the derived integral equation is very simple in form and only requires integration once over the interface elements. Attributed to the feature of being single integral equation, it is easy to adopt the fast multi-pole method to solve large-scale problems [41]. However, the formulations presented in Ref. [34] can only solve linear steady heat conduction problems, and only piece-wise homogeneous materials are analyzed in the adopted numerical examples.

In this paper, a new single integral equation method is developed for solving general multi-medium transient heat conduction problems. Firstly, the boundary-domain integral equation for single medium non-homogeneous transient heat conduction is established. Then from the established integral equation, the interface integral equation for multi-medium transient heat conduction problems is derived, by a degeneration technique from a domain integral to an interface integral. The new formulation allows the thermal material properties (i.e., the thermal conductivity, specific heat and mass density) varying spatially within each medium, and jump across the interfaces between two adjacent different media. For the first time, a single integral equation method is employed to solve multi-medium transient heat conduction problems with variable material properties.

To solve the time-dependent system of differential equations, the finite difference method (FDM) is used in the discretization of time to approximate the time evolution of physical variables. Based on an implicit backward differentiation scheme, an unconditionally stable and non-oscillatory time marching solution scheme is developed for solving the normal time-dependent system of equations, in which only temperature is involved as the time-dependent unknown variable. Numerical examples are given to verify the correctness of the presented method. The results show that, the presented formulations are robust in solving transient heat conduction in multi-medium functionally graded materials.

2. Review of boundary-domain integral equation for transient heat conduction in single non-homogeneous medium

In the extensively used radial integration BEM for solving transient heat conduction in single medium [13], the thermal conductivity k is assumed to be a function of the spatial coordinates $\mathbf{x}$, i.e. $k(\mathbf{x})$. In this paper, not only the thermal conductivity, but also the specific heat c_p and mass density ρ are assumed to be functions of $\mathbf{x}$, i.e. $k(\mathbf{x})$, $c_p(\mathbf{x})$, $\rho(\mathbf{x})$. In this case, the governing equation for transient heat conduction problems can be written as follows:

$$\nabla[k(\mathbf{x})\nabla T(\mathbf{x},t)] + Q(\mathbf{x}) = \rho(\mathbf{x})c_p(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial t} \quad (t > t_0, \mathbf{x} \in \Omega) \tag{1}$$

where $T(\mathbf{x},t)$ is the temperature at location $\mathbf{x}$ at time t; $Q(\mathbf{x})$ is the heat generation; t_0 is the initial time, and Ω represents the computational domain.

The initial condition is

$$T(\mathbf{x},0) = T_0(\mathbf{x}) \tag{2}$$

where $T_0(\mathbf{x})$ is the initial temperature. On the boundary, Dirichlet and Neumann boundary conditions are prescribed as follows:

$$T(\mathbf{x},t) = \overline{T}(\mathbf{x},t), \quad \mathbf{x} \in \Gamma_T \tag{3}$$

$$q(\mathbf{x},t) = -k(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial n} = \overline{q}(\mathbf{x},t), \quad \mathbf{x} \in \Gamma_q \tag{4}$$

where $q(\mathbf{x},t)$ is the normal heat flux on the boundary Γ of the computational domain Ω; n is the unit outward normal to Γ; and $\Gamma = C(\Gamma_T \cup \Gamma_q) = \partial\Omega$, $\Gamma_T \cap \Gamma_q = \varnothing$. In Eqs. (3) and (4), $\overline{T}(\mathbf{x},t)$, $\overline{q}(\mathbf{x},t)$ are the given temperature and heat flux on the boundary, usually prescribed as given functions.

Taking the fundamental solution for the Laplace equation as the weight function, applying the weighted residual technique to Eq. (1), and using the Gauss' divergence theorem, the