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Three-dimensional numerical modeling of free convection in sloping porous enclosures

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Three-dimensional (3D) numerical simulations are carried out to study steady state free convection in a sloping porous enclosure heated from below. The model is based on Darcy's law and the Boussinesq approximation. Two different approaches to solve this problem are compared: primitive variables and vector potential. Although both numerical models lead to equivalent results in terms of the Nusselt number and convective modes, the vector potential model proved to be less mesh-dependent and also a faster algorithm. A parametric study of the problem considering Rayleigh number, slope angle and aspect ratio showed that convective modes with irregular 3D geometries can develop in a wide variety of situations, including horizontal porous enclosure at relatively low Rayleigh numbers. The convective modes that have been described in previous 2D studies (multicellular and single cell) are also present in the 3D case. Nonetheless the results presented here show that the transition between these convective modes follows an irregular 3D geometry characterized by the interaction of transverse and longitudinal coils.

1. Introduction

The problem of free convection in porous media has been of great interest in research due to the widespread presence of this mode of heat transfer in both nature and engineering processes. Geothermal energy and ground water modeling are examples of the application fields of this topic. The problem of a porous enclosure heated from below has been of particular interest for the study of heat transfer rate and steady state convective modes under different parametric conditions. The aim of this paper is to present steady state solutions of free convection in sloping porous enclosures for a range of governing parameters (aspect ratio, slope angle and Rayleigh number) as well as discussing the 3D convective modes present in the parameter space. The steady convection is obtained from the solution of the transient governing equations for long simulation time.

Fundamental aspects of this problem are given by the solution of the Horton–Rogers–Lapwood problem [1]. The solution to this problem establishes the conditions for the onset of convection in a horizontal porous layer heated from below. The early works by Horton and Rogers [2] and Lapwood [3] determined a critical Rayleigh number ($Ra_c = 4\pi^2$) for the onset of convection in such a system. Elder [4] presented one of the first numerical and experimental studies of steady state convection in a two-dimensional (2D) porous enclosure. He described the steady state cellular motion of the fluid, incorporating edge-effects of the porous cavity. Bories and Combarnous [5] extended the analysis to a sloping porous enclosure in 3D following an experimental and theoretical approach. They observed three different kinds of convective regimes, dependent on the model parameters: polyhedral cells similar to the Benard–Rayleigh cells for small slope angles ($\sim 15^\circ$), longitudinal coils (with axis parallel to the longest side of the box) and unicellular flow (which is a 2D velocity distribution) for nearly vertical positions. Regarding the possible convective modes in a horizontal porous enclosure, Holst and Aziz [6] presented one of the earliest numerical models to study this problem in 3D. Considering a set of aspect ratios of a horizontal porous enclosure they determined the possible convective modes for several Rayleigh numbers. They pointed out that as the 2D motion always satisfies the governing equations, when 3D steady state is possible, then the problem is characterized by a multiplicity of solutions. In a later 3D study by Schubert and Straus [7] the Rayleigh numbers at which 2D and 3D solutions can be steady were examined for the case of a cubic porous enclosure. Horne [8] emphasized that steady flows do not necessarily maximize the energy transfer. When multiple solutions are possible, these early studies agree on the dependence of the resulting steady flow on the initial conditions of the problem. Caltagirone and Bories [9]

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presented a theoretical and numerical study for a sloping porous box, their results were consistent with the experimental results by Bories and Combarnous [5]. However they also predicted convective regimes characterized by the interaction of longitudinal coils and transverse rolls. More recent research has been carried out by Barletta and Storesletten [10] to study the stability of transverse and longitudinal convective rolls in an inclined porous channel. These authors described the discontinuous nature of the critical Rayleigh numbers as a function of the inclination angle.

Likewise several studies have been carried out in the past to study this problem in 2D. Moya et al. [11] analyzed steady state convection in tilted square and rectangular cavities and the transition between multicellular convective pattern and single cell as the slope angle and Rayleigh–Darcy number were varied, as well as the existence of multiplicity of steady state solutions. Báez and Nicolás [12] studied a wider range of tilt angles and higher Rayleigh numbers as well as several aspect ratios of the porous cavity. They analyzed how the transition angle between single cell and multiple cell is affected by the Rayleigh number. This problem has been further extended to the analysis of entropy generation [13] and also, more recently, to turbulence [14] and non-Darcian effects [15]. Although these recent studies explore new aspects of the physics of the problem, 3D modeling is an important complementary analysis to identify their range of validity. The aim of this work is to illustrate the complexity of the convective modes that can be present in 3D porous enclosures even at low Rayleigh numbers, and to highlight the importance of 3D modeling for a better understanding of this problem in real three-dimensional systems.

2. Problem formulation

The problem consists of a rectangular porous cavity, tilted at an angle α with respect to the horizontal axis (Fig. 1). The porous medium is assumed to be homogeneous and fully saturated. The problem was stated assuming local thermal equilibrium. Fluid flow is described by Darcy's law and buoyancy effects by the Boussinesq approximation. Viscous heat generation is assumed negligible. From these considerations the momentum equation can be stated as follows (the bar notation denotes dimensional variables and operators):

$$\bar{\mathbf{u}} = -\frac{k}{\mu}(\bar{\nabla}\bar{P} - \rho_0 g\beta(\bar{T} - \bar{T}_0)\mathbf{e}) \quad (1)$$

where $k,\ \mu,\ \rho_0,\ \beta$, and g are permeability, viscosity, density of reference, thermal expansion coefficient and gravitational constant, respectively. Likewise $\mathbf{e} = (\sin\alpha, 0, \cos\alpha)$ gives account of the components of the gravity in the system. The energy equation is as follows

$$\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{\mathbf{u}}\cdot\bar{\nabla}\bar{T} = \bar{\nabla}\cdot(\kappa\bar{\nabla}\bar{T}) \quad (2)$$

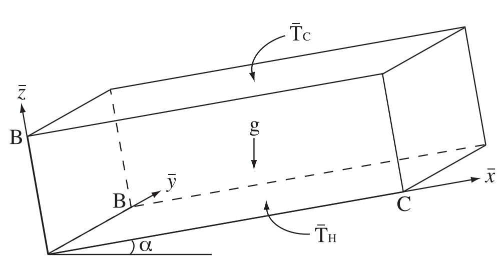

Fig. 1. Schematic model of a sloping porous enclosure heated from below and cooled from the top with adiabatic lateral boundaries.

where κ is the thermal diffusivity. The condition of incompressibility of the fluid is also invoked:

$$\bar{\nabla}\cdot\bar{\mathbf{u}} = 0 \quad (3)$$

Dimensionless variables are defined as follows:

$$x = \frac{\bar{x}}{B} \quad y = \frac{\bar{y}}{B} \quad z = \frac{\bar{z}}{B} \quad P = \frac{k}{\mu\kappa}\bar{P}$$

$$\mathbf{u} = \frac{B}{\kappa}(\bar{u}, \bar{v}, \bar{w}) \quad \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_0 - \bar{T}_c} \quad t = \frac{\bar{t}\kappa}{B^2}$$

$$Ra_p = \frac{Bkg\beta\rho_0}{\kappa\mu}(\bar{T}_0 - \bar{T}_c)$$

where Ra_p is the Darcy–Rayleigh number and B the characteristic length. The dimensionless equations are then as follows, energy equation:

$$\frac{\partial\theta}{\partial t} - \nabla^2\theta + \mathbf{u}\cdot\nabla\theta = 0 \quad (4)$$

The dimensionless momentum equation is as follows:

$$\mathbf{u} + \nabla P = Ra_p\theta\mathbf{e} \quad (5)$$

The domain is given by $0 \leqslant x \leqslant D,\ 0 \leqslant y \leqslant 1,\ 0 \leqslant z \leqslant 1$, with $D = C/B$, the aspect ratio. Additionally, a global Nusselt number is defined to quantify the heat transfer through the upper surface $z = 1$:

$$Nu = \int \left|\frac{\partial\theta}{\partial z}\right|_{z=1} dA \quad (6)$$

2.1. Boundary conditions and initial conditions

It is assumed that the system rests at mechanical and thermal equilibrium as the initial condition. Additionally, the initial dimensionless temperature is set to zero. Assuming that the lateral walls of the cavity are adiabatic $(x = 0, x = D, y = 0, y = 1)$ and the bottom and top boundaries have specified temperatures, the boundary conditions for the energy equation can be written as

$$\frac{\partial\theta}{\partial x} = 0, \quad \text{for } x = 0 \text{ and } x = D$$

$$\frac{\partial\theta}{\partial y} = 0, \quad \text{for } y = 0 \text{ and } y = 1$$

$$\theta = 1, \quad \text{for } z = 0 \text{ and } \theta = 0, \quad \text{for } z = 1 \text{ for } t > 0$$

Regarding the momentum equation impermeable boundary conditions are assumed. The implementation of these boundary conditions is described in the following section.

3. Numerical solution

There are two numerical approaches to solve the problem given above: primitive variables and vector potential. The vector potential approach has been historically preferred [6,8,16,17], since it has proven to be a faster computational algorithm. A comparison of these two methods has not been presented before however.

3.0.1. Primitive variables approach

Taking the divergence of Eq. (5) and considering the incompressibility condition, a Poisson equation for the pressure is obtained