International Journal of Heat and Mass Transfer 97 (2016) 201–210

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Two-equation method for heat transfer efficiency in metal honeycombs: An analytical solution

H. Liu [a], Q.N. Yu [a,*], Z.C. Zhang [a], Z.G. Qu [b], C.Z. Wang [a]

[a] *National Key Laboratory of Human Factors Engineering, Astronaut Center of China, Beijing 100094, China*
[b] *Key Laboratory of Thermal Fluid Science and Engineering of Ministry of Education, School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China*

ARTICLE INFO

Article history:
Received 1 July 2015
Received in revised form 3 November 2015
Accepted 6 January 2016

Keywords:
Metal honeycombs
Two-equation method
Effective medium
Forced convection

ABSTRACT

An analytical model based on the two-equation method is proposed in this study to evaluate the heat transfer performance in sandwiched metal honeycomb heat exchangers under forced convection conditions. The local thermal non-equilibrium between a cooling fluid and solid honeycombs is considered. The validity and the accuracy of the analytical results are verified by numerical simulation. Compared with the convectional corrugated wall, effective medium, and transfer matrix models, the present analytical predictions are closer to the finite element simulation results in a wide range of relative density. According to the analytical solutions, the effects of cell wall length, relative thickness of the heat exchanger, fluid-to-solid thermal conductivity ratio on flow characteristics, and heat transfer performance are subsequently examined to obtain the optimum design for compact heat exchangers that require high heat transfer performance. Results show the mutual influence between cell wall length and relative thickness. The use of a metal honeycomb in compact heat exchangers can significantly enhance heat transfer with a low pressure drop.

1. Introduction

Superior mechanical, heat transfer, and other physical properties, as well as recent advances in low-cost processing, have enabled the wide use of two-dimensional cellular metal prismatic honeycombs as the core of compact heat exchangers that require high heat transfer performance at minimum weight [1]. Continuous channels of this sandwiched prismatic honeycomb structure with a single “easy-flow” direction enable internal fluid transport and thus simultaneous active cooling [2,3]. The surface area per unit volume (i.e., the surface area density or specific surface area) of a typical micro-cell honeycomb is about 3000 $\mathrm{m}^2 \cdot \mathrm{m}^{-3}$, which makes it an ideal material for strengthening heat transfer under a forced convection condition [4].

Many models have been developed in recent years [4–6]. In theoretical studies, three analytical models are used to describe the characteristics of heat transfer in a two-dimensional prismatic honeycomb structure under forced convection conditions. The first one is the corrugated wall model [4,6] that is also called the modified fin analogy model [2]. The corrugated wall method can model the detailed cellular honeycomb structure as a corrugated wall with many fins. The temperature field of a single corrugated wall is solved by excluding the effects of fins, and it is used to decide the heat loss of the corrugated wall. The accuracy of the obtained temperature field is clearly limited by the assumed fin geometry. Therefore, the heat loss of the corrugated wall is approximate, whereas the approximation of heat dissipation to the cooling fluid from the fin attachments overestimates the overall heat transfer of the honeycomb structure. The second one is the effective medium model [6], which is based on a one-equation method assuming a local thermal equilibrium (LTE) of fluid and solid phases. The effective medium model uses the volume averaging technique and the specific surface area used in the governing equations that count the heat transfer enhancement from the corrugated walls and fins simultaneously. However, the different contributions to the heat transfer enhancement from the corrugated wall and fins are ignored, and the fluid temperature at different transverse positions is assumed to be the same, thus leading to an obvious underestimation. The third is the transfer matrix model [5]. Similar to the corrugated wall method, the transfer matrix method can also model the heat flux transfer along a slice of honeycomb in the corrugated walls and fins simultaneously based on the conservation of energy. This model somewhat overcomes the underestimation of the effective medium model and the approximation of heat dissipation contributed by the fin attachments. The computing unit is based on a single cell wall of length and the corresponding fin,

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* Corresponding author. Tel./fax: +86 010 66362260.
E-mail address: yuqingni@139.com (Q.N. Yu).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.01.020
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Nomenclature

a	cell size, mm
c_a	shape factor
c_f	frictional coefficient
c_{ky}	proportionality coefficient for effective thermal conductivity
c_p	specific heat capacity of fluid, $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
D_h	hydraulic diameter
h	local heat transfer coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$
$\bar{h}$	overall heat transfer coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$
H, L, W	thickness, length, and width of the sandwiched heat exchanger, respectively, mm
I	thermal performance index
k_s, k_f	thermal conductivity of solid and fluid, $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$
l	cell wall length, mm
$\dot{m}$	mass flow rate, $\text{kg} \cdot \text{s}^{-1}$
N_s	total number of slices over width W
Nu	Nusselt number
Δp	pressure drop, Pa
q	heat flux, $\text{W} \cdot \text{m}^{-2}$
Re	Reynolds number
t	cell wall thickness, mm
$T, \bar{T}$	temperature and its average, K
u	velocity, $\text{m} \cdot \text{s}^{-1}$
(x, y, z)	global Cartesian coordinates

Greek symbols

α_a	specific surface area, m^{-1}
ε	porosity
μ	dynamic viscosity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
ν_f	kinematic viscosity, $\text{m}^2 \cdot \text{s}^{-1}$
ρ_f	density of fluid, $\text{kg} \cdot \text{m}^{-3}$
ρ	relative density of the honeycomb structure

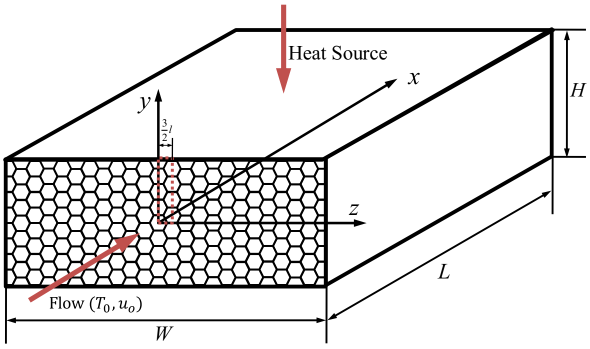
Subscripts

e	effective
f	fluid
s	solid
w	substrate wall
0	inlet

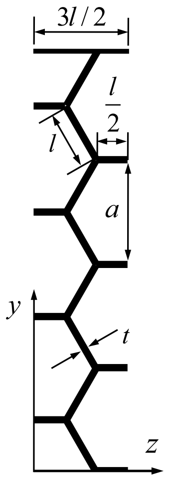
whereas, the energy equations for the solid and fluid phases are difficult to solve analytically if the transverse fluid temperature gradient is considered. In addition, the transfer matrix method still shows various less accurate predictions in a wide range of relative density of the honeycomb structure compared with the simulation result.

In general, the temperature gradient through a transverse direction is ignored for the three aforementioned models. The fluid temperature field distribution is assumed to be uniform and is represented by the mean fluid temperature. The local thermal equilibrium of solid and fluid phases is specifically assumed in these studies. The so-called one-equation model associated with LTE can adequately describe the heat transfer characteristic in a porous medium when the temperature difference between the solid and fluid phases is negligible. In reality, the fluid temperature near the bottom- and the top-heated substrate surface is significant when the thermal conductivity ratio between solid and fluid is high enough. The one-equation model can lead to a significantly diverse convective heat transfer efficiency between the wall (or fin) and the cooling fluid in a different transverse position. Therefore, the assumption of having an LTE is questionable. The local thermal non-equilibrium (LTNE) model is more accurate than the LTE model, as indicated by Lee and Vafai [7]. The LTNE model has been adopted by many researchers to represent the fluid–solid energy exchange [8–10] in porous media. Therefore, the LTNE model is required, and the analytical model for the heat transfer of metal honeycomb structures can be further improved.

In this study, the transverse fluid temperature gradient is considered using the two-equation method. An analytical model is presented to analyze the heat transfer in a sandwiched metal honeycomb structure based on the two-equation effective medium model that uses volume-averaging techniques. Exact solutions of the overall heat transfer coefficient and the dimensionless thermal performance index are obtained. New analytical solutions are validated by finite element simulation results in a wide range of relative density of the honeycomb structure. The effects of cell wall length, relative thickness, fluid-to-solid thermal conductivity ratio in flow characteristics, and heat transfer performance are subsequently examined.



(a) The global coordinate



(b) A slice of the heat exchanger

Fig. 1. A prototypical design of the heat exchanger cooled by forced convection through metal honeycomb: (a) the global coordinate; (b) a slice of the heat exchanger.