International Journal of Heat and Mass Transfer 97 (2016) 353–363

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Linear stability analysis of thermocapillary flow in a slowly rotating shallow annular pool using spectral element method

Linmao Yin [a,b], Zhong Zeng [a,c,d,*], Zhouhua Qiu [a], Huan Mei [a], Liangqi Zhang [a], Yongxiang Zhang [a,d]

[a] Department of Engineering Mechanics, College of Aerospace Engineering, Chongqing University, Chongqing 400044, China
[b] State Key Laboratory of Coal Mine Disaster Dynamics and Control (Chongqing University), China
[c] State Key Laboratory of Crystal Material (Shandong University), China
[d] Chongqing Key Laboratory of Heterogeneous Material Mechanics (Chongqing University), China

ARTICLE INFO

Article history:
Received 15 June 2015
Received in revised form 1 February 2016
Accepted 1 February 2016

Keywords:
Thermocapillary flow
Spectral element method
Linear stability analysis
Energy analysis

ABSTRACT

The stability of thermocapillary flow in a slowly rotating shallow annular pool was investigated by using the Legendre spectral element method. The silicon melt (Pr = 0.011), filling in a pool with adiabatic free surface and bottom, was heated at the outer cylindrical wall and cooled at the inner cylindrical wall. The critical stability conditions for different dimensionless rotation rates ω, ranging from 0 to 2000, were determined by linear stability analysis. Moreover, the energy analysis was applied to further illustrate the underlying mechanism of the flow instability. The results indicate that there is one Hopf bifurcation for $\omega < 940$ and $\omega > 1185$. Thermocapillary flow is the dominant factor of the instability for $\omega < 940$, and the pool rotation becomes the key role for the first instability for $\omega > 1185$. Three turning points were observed in the interval $940 \leqslant \omega \leqslant 1185$, corresponding to three transitions between two-dimensional steady flow and three-dimensional oscillatory flow, owing to the competition of two driving forces with increasing Marangoni number at a fixed ω. With pool rotation, the results exhibit that the flow instability is deduced to occur initially in the zone near the cold wall with the evidence of the extreme velocity gradient and also the distribution of the local kinetic energy.

1. Introduction

Thermocapillary flow, originating from the unbalanced surface tension, plays an important role in materials processing applications, for instance, crystal growth from the melt under microgravity. The instability of the thermocapillary flow has attracted an increasing attention, and therefore, a large number of experiments, numerical simulations and linear stability analyses about the thermocapillary flow in annular liquid pools had been performed in the past few decades.

Schwabe et al. [1] conducted experiments on the thermocapillary flow in an annular gap with thickness of 0.6–3.6 mm (inner radius $r_i = 20$ mm and outer radius $r_o = 77$ mm) by using ethanol (Prandtl number Pr = 17). They observed various instabilities and they found gravity could be negligible when the thickness is less than or equal to 3.0 mm. Aboard the Space Shuttle, Kamotani et al. [2] also performed many experiments of oscillatory thermocapillary flow in open circular containers with different inner diameters (12, 20, and 30 mm). They observed two lobed surface temperature patterns and measured the critical heat fluxes in the tests with container aspect ratio being equal to 1. In addition, Mukolobwiez et al. [3] observed hydrothermal waves (HWs) traveling in the azimuthal direction by using silicone oil (Pr = 10), and the same phenomenon was also reported by Garnier et al. [4]. Schwabe et al. [5] used the 0.65 cSt silicone oil (Pr = 6.8) to study thermocapillary flow in open annuli with the liquid height from 1.25 mm to 20 mm ($r_o = 40$ mm and $r_i = 20$ mm) under microgravity. They observed HWs traveling on the free surface for thin annular gaps and determined the critical parameters for the incipience of oscillations under various conditions.

Sim and Zebib [6] simulated the three-dimensional thermocapillary flow in an open cylindrical annulus with aspect ratio being equal to 1. They observed two or three azimuthal waves under microgravity with a rotating annular pool. Subsequently, Sim et al. [7] conducted numerical simulations of thermocapillary flow in an annular pool of silicone oil with the same parameters as in Schwabe's microgravity experiments [5], and they stated that the numerical results agreed well with the experiments. Shi and Imaishi [8] simulated three-dimensional thermocapillary flow

* Corresponding author at: Department of Engineering Mechanics, College of Aerospace Engineering, Chongqing University, Chongqing 400044, China. Tel.: +86 23 65111813.
E-mail address: zzeng@cqu.edu.cn (Z. Zeng).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.02.031
0017-9310/

(Pr = 6.7) in a static shallow annular pool model. They reported that the wave number and angular velocity of hydrothermal wave varied with Marangoni number. Later on, Li et al. [9] presented three-dimensional thermocapillary and thermocapillary-buoyancy flow in annular pool of silicon melt (Pr = 0.011). Slowly rotation rate from 0 to 2 rpm of annular pool was considered, and two kinds of hydrothermal wave were observed as well.

In comparison to the experiments and the direct numerical simulations, the linear stability analysis (LSA) is a more efficient and timesaving strategy to detect the onset of the instability. Smith and Davis [10,11] applied the LSA to a thin and infinitely extended fluid layer with a free surface subjected to a horizontal temperature gradient. They reported two instabilities with the corresponding critical Marangoni numbers. By using the LSA, Zebib's [12] presented that the system rotation affected the instabilities of thermocapillary flow under microgravity, and they claimed that the Coriolis force should be considered. Shi et al. [13] studied the stability of the thermocapillary flow (Pr = 6.7) in a shallow annular pool (r_i = 20 mm, r_o = 40 mm and depth d = 1 mm), and they reported that the pool rotation destabilized the steady axisymmetric thermocapillary flow and increased the critical azimuthal wave numbers. Li et al. [14] applied the LSA to a static annular pool in order to understand the stability characteristics of the thermocapillary flow with Pr = 0.011 fluid. They investigated the influence of the gravity and different aspect ratios on the critical Marangoni number and wave number.

The LSA is an effective measure to study the onset of the flow instability. However, capturing accurately the critical conditions relies heavily on the accuracy of the numerical methods, and therefore, a high-order accurate numerical method is necessary in the LSA [15,16]. The spectral element method (SEM), combining spectral method with the high precision and finite element method with the geometrical adaptability, was proposed by Patera [17] in 1984. With the development of the SEM in computational fluid dynamics [18–20], SEM is inherently a good choice in the LSA, and the practicability for applying SEM to LSA was proved [21,22].

In this paper, a Legendre SEM was adopted to simulate the thermocapillary flow in a rotating annular pool, the LSA was applied to determine the critical condition for the onset of the instability, and the instability mechanism were investigated by the energy analysis.

2. Problem description

As is exhibited in Fig. 1, silicon melt with top free surface is filled in a rotating annular pool with the inner radius r_i = 15 mm, outer radius r_o = 50 mm and depth d = 3 mm. The outer cylindrical wall is maintained at a constant temperature Φ_h while the inner cylindrical wall is at a lower temperature Φ_c, and the temperature difference $\Delta\Phi$ is defined as $\Delta\Phi = \Phi_h - \Phi_c$. The top free surface of the melt is idealized as a non-deformable surface, and the surface tension on the free surface is considered to be a linearly decreasing function of the temperature as $\sigma = \sigma_0 - \gamma_T\Phi$, where the surface tension coefficient is defined as $\gamma_T = -\partial\sigma/\partial\Phi$. In addition, the silicon melt is taken as an incompressible Newtonian fluid and its physical properties are listed in Table 1.

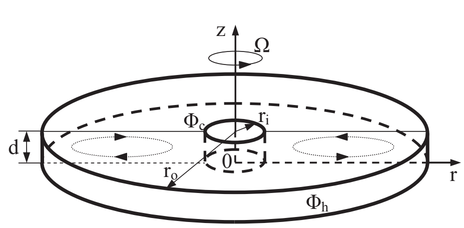

Fig. 1. The geometrical model.

In this study, the convection is driven by the unbalanced surface tension on the free surface and the rotation of annular pool. Taking the characteristic scales of time $(r_o - r_i)^2/\upsilon$, length $r_o - r_i$, velocity $\upsilon/(r_o - r_i)$ and pressure $\mu\upsilon/(r_o - r_i)^2$, the dimensionless Navier-Stokes equations in zero gravity condition are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nabla^2 \mathbf{u}, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T = \frac{1}{\mathrm{Pr}}\nabla^2 T, \tag{2}$$

$$\nabla \cdot \mathbf{u} = 0. \tag{3}$$

Here, $\nabla = \partial_{,r}\mathbf{i}_r + (\partial_{,\theta}/r)\mathbf{i}_\theta + \partial_{,z}\mathbf{i}_z$. $\mathbf{u}$ with $\mathbf{u} = u\mathbf{i}_r + v\mathbf{i}_\theta + w\mathbf{i}_z$ is the dimensionless velocity vector. r, θ, z represent radial, azimuthal, axial directions with the unit vector $\mathbf{i}_r$, $\mathbf{i}_\theta$, $\mathbf{i}_z$, respectively. t denotes the dimensionless time. p denotes the dimensionless pressure. $T = (\Phi - \Phi_c)/\Delta\Phi$ is the dimensionless temperature. The Prandtl number Pr is defined as $\mathrm{Pr} = \upsilon/\kappa$.

With the definition of the aspect ratio $\varepsilon = d/(r_o - r_i) = 0.086$ and the radius ratio $\beta = r_i/r_o$, the boundary conditions are expressed as below:

the free surface ($z = \varepsilon$)

$$\frac{\partial u}{\partial z} = -\frac{\mathrm{Ma}}{\mathrm{Pr}}\frac{\partial T}{\partial r},\ \frac{\partial v}{\partial z} = 0,\ w = 0,\ \frac{\partial T}{\partial z} = 0, \tag{4}$$

the bottom ($z = 0$)

$$u = 0,\ v = r\omega,\ w = 0,\ \frac{\partial T}{\partial z} = 0, \tag{5}$$

the inner cylinder wall ($r = \beta/(1 - \beta)$)

$$u = 0,\ v = \frac{\beta}{(1-\beta)}\omega,\ w = 0,\ T = 0, \tag{6}$$

the outer cylinder wall ($r = 1/(1 - \beta)$)

$$u = 0,\ v = \frac{1}{(1-\beta)}\omega,\ w = 0,\ T = 1. \tag{7}$$

Here, the Marangoni number Ma is defined as $\mathrm{Ma} = \gamma_T\Delta\Phi(r_o - r_i)/(\mu\kappa)$. The dimensionless rotational angular velocity ω is defined as $\omega = \Omega(r_o - r_i)^2/\upsilon$, where Ω as in Fig. 1 is the angular velocity of the pool rotation.

3. Numerical solution techniques

The adopted SEM was introduced in this section. The non-staggered grids based on the Legendre–Gauss–Lobatto points were applied. The properties of Legendre polynomial and Legendre expansions were presented in our previous paper [23].

3.1. Basic flow

The time splitting method [24,25] is applied in simulating two-dimensional axisymmetric basic flow, and the semi-discrete

Table 1
Physical properties of silicon melt.

Item	Value	Unit
Prandtl number, Pr	0.011	–
Density, ρ	2530	kg/m^3
Surface tension coefficient, γ_T	7×10^{-5}	N/(m K)
Thermal diffusivity, κ	2.53×10^{-5}	m^2/s
Kinematic viscosity, υ	2.767×10^{-7}	m^2/s
Dynamic viscosity, μ	7×10^{-4}	kg/(m s)