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Nonlinear thermal parameter estimation for embedded internal Joule heaters

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ABSTRACT

We propose a novel inverse scheme, which allows for estimation of thermal parameters of internal Joule heaters through measurements of surface temperature distributions during a Joule heating process. The inverse scheme is based on the governing nonlinear, inhomogeneous heat conduction and generation equation and solely assumes knowledge of the electric resistivity of the Joule heater. Polynomial forms are assumed for the thermal conductivity $\kappa = \kappa(T)$ and $c_p\rho =: \lambda = \lambda(T)$, while the method can be easily generalized to estimate parameters of any suitable form. Both the sensitivity and the adjoint methods are developed and compared. Owing to the ill-conditioning of the inverse scheme, the performance of relaxation methods and regularization schemes are analyzed (to improve numerical conditioning). A verification was conducted using polydimethylsiloxane (PDMS) embedded with a strip of conductive propylene-based elastomer (cPBE). Good agreement was achieved between theoretical predictions by the inverse scheme and experimental measurements regardless of the approximated effective potential difference across the cPBE. While constant parameter estimations sufficed to approximate one reference temperature, the inclusion of multiple instants of time required an increase in the polynomial order. The improved parameter estimation is shown to remain of the same order of magnitude for the temperature range encountered when compared with the constant approximation, i.e. $\kappa = 10.7$ and $12.0\ \mathrm{W\,m^{-1}\,K^{-1}}$, and $\lambda = 19.9$ and $16.2\ \mathrm{J\,m^{-1}\,K^{-1}}$, respectively.

1. Introduction

Phase change and glass transition are typically used to enable rapid changes in the elastic rigidity of soft bio-inspired systems. Examples include PVAc–nanowhisker composites [1], shape memory polymer [2], wax [3], and conductive propylene-based elastomer (cPBE) composed of a percolating network of CB microparticles in a propylene–ethylene co-polymer [4]. In the case of cPBE, electrical current is applied to heat the composite above its glass transition temperature and to induce mechanical softening. Because of the low glass transition temperature T_g (~75 °C), activation can be achieved within a few seconds. However, further progress depends on improved electro-thermo-elasto characterization of the composite and surrounding insulating materials. Knowing parameters like specific heat c_p, density ρ and thermal conductivity κ allows for predictive modeling that can be used to identify material compositions and geometries that reduce electrical power requirements and activation time.

Here, we examine the thermal response of a general case of elastomeric composites illustrated in Fig. 1. The thermal phenomena under investigation are governed by the following transient heat conduction equation:

$$\rho(T)c_p(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left\{\kappa(T)\frac{\partial T}{\partial x}\right\} + q(T), \tag{1}$$

where T denotes temperature and $q(T)$ denotes the voltage dependent nonlinear heat generation and x denotes the spatial n-dimensional variable. We derive the adjoint equation for this method and compare it with the sensitivity-matrix-based approach. For the latter, different regularisation schemes are analyzed in order to improve the conditioning of the inversion of the sensitivity matrix. Relaxation methods are analyzed for both approaches. In order to obtain solutions for the direct scheme, a hybrid finite

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difference method (FDM) is applied, combining the merits of both implicit and explicit FDMs.

As shown in Fig. 1, the material of interest is embedded in PDMS, which has known physical parameters for the temperature range considered in this study. We will demonstrate that by embedding the material in PDMS, the precision of the emissivity matrix as recorded by a thermal microscope *Infrascope* (see Fig. 2b) can be considerably augmented. Internal Joule heating is presented as a novel tool to permit inverse determination of the thermal parameters of the material under investigation and possibly their temperature dependence. It is shown that this is only permissible for small enough heat generations, since otherwise, diffusion effects are difficult to capture and thus the estimation of such parameters becomes highly ill-conditioned.

Section 2 presents previous achievements in the field of inverse schemes particularly with respect to heat conduction as well as the motivation behind the novel inverse scheme. The subsequent section treats the underlying theory of the inverse scheme, including a discretized version of the governing PDE, the sensitivity and adjoint method and methods to improve the conditioning of the inherently ill-conditioned inverse scheme. A method to experimentally validate the applicability of the inverse scheme is given in Section 4 alongside the physical theory on which it is based. In Section 5, the inverse scheme's performance is first assessed in a theoretical one-dimensional framework, before the two-dimensional case is analyzed. These results are also compared with results from the experimental validation section. Finally, conclusions together with suggestions on how the scheme can be improved in the future is finally given in Section 6.

2. Background

Estimation of thermal parameters for new, sparsely studied materials is vital for numerous applications. Historically, these include space-related problems [5], the testing of components used in nuclear reactors [6,7] and temperature control for heat-treatment-based manufacturing processes such as quenching [8]. A more specific field of application consists in the control of the boundary temperature around Joule heaters, as depicted in [4] to allow close interaction with human skin. While for some of these applications it suffices to find a constant value invariant of space, time or any thermodynamic state, other materials show non-linear behavior and hence complicate the search for these properties. While the determination of temperature distribution via knowledge of the governing parameters is referred to as the *direct problem*, the estimation of parameters via knowledge of the temperature distribution at discrete time points on a subset of the spatial domain is termed the *inverse problem* or alternatively, in the specific case of thermal conduction dominated environments, the *inverse heat conduction problem* (IHCP). Since small variations in the observed data, i.e. the temperature distribution on parts of the spatial domain, can lead to large variations in the estimation of the parameters, the inverse problem is ill-conditioned in essence. Noise within the given data is therefore unfavorable and to be kept at a minimum, as far as the nature of the experimental apparatus and its inherent imprecision permit.

Throughout the past decades, numerous inverse schemes have been proposed, which vary both in the methods applied as well as the restrictions and limitations imposed on the model. A fundamental categorization classifies schemes as either stochastic (e.g. [9–11]) or gradient based (e.g. [12–15]). This paper concentrates on the latter, of which the utmost part, as the name indicates, is based on an iterative process, in which the estimated parameters are steadily improved by determining the gradient and starting from an initial, to a certain degree arbitrary guess. The majority of methods can be categorized with respect to the means employed to obtain the gradient. The sensitivity method (e.g. [16–19]) is based on a numerical determination of the dependence of an incremental change in one of the parameters on the temperature distribution and thus evaluates the optimal gradient. The system of equations to be solved is ill-conditioned, but can be regularized using schemes such as L^2-regularization [20], Tykhonov-L^1 [21] or TSVD [22]. An alternative is given by the so-called adjoint method, which is based on the solution of the adjoint equation of the governing partial differential equation [23]. Even if the direction of the gradient obtained using one of the above methods reliably indicates the direction of steepest descent and hence an improved agreement between observation and simulation, the gradient's magnitude generally remains to be optimized. Although the scaling is usually highly case-specific, established methods exist, which hold across a diverse field of applications (see e.g. [24]). Methods differ in the assumption of the representation of the parameters. While functions free of assumptions on the functional form exist, other approaches assume polynomial or other suitable functional forms. The dependence itself further varies from case to case in that the independent variables comprise space or time and in other cases temperature itself.

Although numerous methods solving inverse problems based on reference data on the boundary have been presented throughout the past few decades (see e.g. Alfanov et al. [25]), a significantly smaller number of investigations addressing the experimental means of obtaining the boundary temperature is currently present. A classical means consists in simply heating one of the surface by a given heat flux and then determining the parameters from measurements on the boundary, as presented by Beck [26]. Hon [27] as well as Alifanov [12] investigated the IHCP involving the estimation of a boundary flux given the temperature distribution on a subset of the spatial domain. The case in which temperature measurements on the object are impractible is addressed by Howell [28], who uses remote measurements to apply an inverse method to estimate the radiative thermal properties. Owing to the ill-conditioned nature of the IHCP, issues with the precision of the transferred energy via external means can arise. This includes, for example, the extent to which hotplates can maintain a certain temperature to a constant level.

3. Theory on direct and inverse schemes

3.1. Finite difference scheme for nonlinear diffusion equations

Since for simple geometries, finite difference-based schemes provide sufficient accuracy together with low computational cost, the gain in accuracy via finite volume schemes, both fixed in order or of arbitrary order of accuracy, does not stand in relation to the increase in computational cost. The main challenge in constructing a stable and accurate scheme for the nonlinear heat-transport equation presented in Eq. (1) lies in the temperature-dependence of the parameters. While both transient and generation terms can be discretized at the $N_x + 1$ nodes of a grid $\tau_n = \{x_i \,|\, i \in \{0, 1, \dots, N_x + 1\} \subset \mathbb{R}, N_x \in \mathbb{N}$, where x_i denotes the position of node i, the conduction term requires the introduction of an additional interface grid $\tau_i = \{x_{i-1/2} \,|\, i \in \{0, 1, \dots, N_x + 2\}\} \subset \mathbb{R}$ with $x_{min} = x_{-1/2} = x_0 < x_1 < x_{3/2} < \dots < x_N < x_{N+1/2} < x_{N+1} = x_{N+3/2} = x_{max}$, $(x_{min}, x_{max}) \in \mathbb{R}^2$. The full grid containing both nodes and interfaces is denoted $\tau := \tau_n \cup \tau_i$.

In favor of a generalized approach, allowing explicit, semi-implicit as well as implicit treatments of the time-derivative, the following scheme is introduced [29]: