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Influences of an effective Prandtl number model on nano boundary layer flow of γ Al_2O_3–H_2O and γ Al_2O_3–$C_2H_6O_2$ over a vertical stretching sheet

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ABSTRACT

Nanoparticles provide potentials in augmenting the performance of convective heat transfer in the boundary layer flow region. Prandtl number plays a vital role in controlling the momentum and thermal boundary layers. In view of this, the influences of an effective Prandtl number model which is derived from experimental data (Pop et al., 2007) on the nano boundary layer, steady, two-dimensional and laminar flow of an incompressible γ Al_2O_3–H_2O and γ Al_2O_3–$C_2H_6O_2$ nanofluids over a vertical stretching sheet are investigated for the first time in the present article. The models which are used for viscosity and thermal conductivity also derived from experimental data (Maiga et al., 2004a; Maiga et al., 2005). The second law of thermodynamics also analysed for the present problem. The transformed governing nonlinear boundary layer equations are solved numerically using fourth order Runge–Kutta method with shooting technique and analytical solutions are presented for a special case. The numerical results obtained for the temperature profile, skin friction coefficient and reduced Nusselt number are presented through plots for two different cases such as with and without effective Prandtl number. It is found that the increasing values of nanoparticle volume fraction of γ Al_2O_3 nanoparticles decrease the temperature of the nanofluids in the presence of effective Prandtl number and increase in the absence of effective Prandtl number. The entropy generation number is higher for ethylene glycol based nanofluids compared to water based nanofluids.

1. Introduction

In many industrial processes such as cooling of metallic sheets, crystal growing, manufacture and drawing of plastics and rubber sheets, glass-fibre and paper production, metal and polymer extrusion processes, the problem of flow and heat transfer induced by a stretched surface is very important. The rate of cooling plays an important role about the quality of the final product obtained from these processes, in which a moving surface emerges from a slit, and as a consequence, a boundary layer flow is appeared in the direction of the movement of the surface. Crane [1] studied the steady two dimensional boundary layer flow of Newtonian fluid over a stretching surface. After this pioneering work the flow field over a stretching surface has gain considerable attention and a good amount of literature has been generated on this problem [2–4].

Suspensions of nanometer sized particles in conventional heat transfer fluid are called nanofluids [5] which are suitable for engineering applications and show several potential advantages such as better stability, high thermal conductivity and no extra pressure drop compared to other suspensions. Due to the exclusive features of nanofluids, many researchers have been motivated to investigate the heat transfer performance and flow characteristics of various nanofluids with different nanoparticles and base fluid materials [6–8]. Alumina nanofluids have attracted the research community because of its applications in many cooling processes [9–14]. Alumina nanoparticles are classified according to their size as alpha alumina and gamma alumina etc. Surface properties of well-characterized samples of eta and gamma alumina were studied in [15].

The boundary layer flow of nanofluids over a stretching surface has been considered by many researchers in recent years [16–28] with various metal and oxide nanoparticles. Hajmohammadi et al. [29] investigated the water based copper and silver

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Nomenclature

g	acceleration due to gravity
T	temperature of the nanofluid
T_w	temperature of the nanofluid on the wall
T_∞	ambient temperature
Pr_{nf}	Prandtl number of the nanofluid
Pr_f	Prandtl number of the base fluid
$\frac{1}{2}\mathrm{Re}_x^{1/2}C_f$	local skin friction coefficient
$\mathrm{Re}_x^{-1/2}Nu_x$	reduced Nusselt number
k_{nf}	thermal conductivity of the nanofluid
k_f	thermal conductivity of the base fluid
k_s	thermal conductivity of the nanoparticles
u,v	velocity components in x and y directions, respectively
ϕ	nanoparticle volume fraction
λ	mixed convection parameter
ρ_{nf}	effective density of the nanofluid
ρ_f	density of the base fluid
ρ_s	density of the nanoparticles
$(\rho\beta)_{nf}$	thermal expansion coefficient of the nanofluid
$(\rho\beta)_f$	thermal expansion coefficient of the base fluid
$(\rho\beta)_s$	thermal expansion coefficient of the nanoparticles
μ_{nf}	effective dynamic viscosity of the nanofluid
μ_f	dynamic viscosity of the base fluid
η	space variable

nanofluids over a stretching surface. Very recently, Vishnu Ganesh et al. [30] studied the boundary layer flow of alumina and gamma alumina nanofluids over a horizontal stretching sheet without considering effective Prandtl number.

The second law of thermodynamics is more reliable than the first law of thermodynamics because of the limitation of the efficiency of the first law in heat transfer engineering systems. In order to access the best design of thermal systems, one can employ the second law of thermodynamics by minimizing the irreversibility [31,32]. The greater entropy generation implies the greater extent of irreversibilities. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process. Likewise, it can be used to establish criteria for the performance of engineering devices. Entropy generation is a criterion of the destruction of the available system work. The evaluation of the entropy generation is carried out to improve system performance [33–39]. Heat mass transfer, viscous dissipation, etc. can be used as sources of entropy generation. Rashidi et al. [40] analysed the entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid All the above mentioned studies have reported the nanofluid boundary layer flow without considering any effective Prandtl number.

A close observation of the literature reveals that, to the best of author's knowledge, so for no one has considered the influences of an experimental based Prandtl number model on the boundary layer flow of γ Al_2O_3–H_2O and γ Al_2O_3–$C_2H_6O_2$ over a vertical stretching sheet. Keeping this in mind, in the present article, we investigated the boundary layer flow of steady two-dimensional laminar flow of an incompressible γ Al_2O_3–H_2O and γ Al_2O_3–$C_2H_6O_2$ nanofluids over a vertical stretching sheet in the presence of an effective Prandtl number model. The viscosity, thermal conductivity and an effective Prandtl number models which are derived from experimental data [9,11,12] used for the present study. Numerical solutions are obtained for the governing transformed equations using fourth order Runge–Kutta method with shooting technique. Analytical solutions are presented for a special case. The mathematical formulation of problem is given in the next section.

2. Formulation of the problem

Consider a steady two-dimensional laminar boundary layer flow of an incompressible γ Al_2O_3–H_2O and γ Al_2O_3–$C_2H_6O_2$ nanofluids over an impermeable stretching sheet. We select a coordinate frame in which x-axis is aligned vertically upwards. The nanofluid flow is generated, due to the stretching of the sheet, caused by two equal and opposite forces along the x -axis. The velocity of the stretching sheet is $u_w = ax$ and the temperature at the stretching surface is $T_w = T_\infty + bx$. Where a and b are constants and T_∞ is the ambient temperature. It is also assumed that the base fluids and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluids are considered as in Table 1. Taking the above assumptions into consideration, the steady boundary layer equations governing the convective flow and heat transfer for a nanofluid can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}+g\frac{(\rho\beta)_{nf}}{\rho_{nf}}(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The corresponding boundary conditions are

$$u=u_w,\quad v=0,\quad T=T_w\,(T_w=T_\infty+bx)\quad at\quad y=0,$$
$$u\to 0,\quad T\to T_\infty\quad \text{as}\quad y\to\infty,$$

where u and v are the velocity components along the axis x and y, respectively, g is the acceleration due to gravity, T is the temperature of the nanofluid, T_∞ is the temperature of the nanofluid far away from the wall.

Table 1
Thermo physical properties of water, ethylene glycol and alumina.

	ρ (kg/m^3)	C_p (J/kg K)	k (W/m K)	$\beta\times10^{-5}$ (K^{-1})	Pr
Pure water (H_2O)	998.3	4182	0.60	20.06	6.96
Ethylene glycol ($C_2H_6O_2$)	1116.6	2382	0.249	65	204
Alumina (Al_2O_3)	3970	765	40	0.85	–