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Analytical solution of non-stationary heat conduction problem for two sliding layers with time-dependent friction conditions

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ABSTRACT

In this article we conduct an overview of various types of thermal contact conditions at the sliding interface. We formulate a problem of non-stationary heat conduction in two sliding layers with generalized thermal contact conditions allowing for dependence of the heat-generation coefficient and contact heat transfer coefficient on time. We then derive an analytical solution of the problem by constructing a special coordinate integral transform. In contrast to the commonly used transforms, e.g. Laplace or Fourier transforms, the one proposed is applicable to a product of two functions dependent on time. The solution is validated by a series of test problems with parameters corresponding to those of real tribosystems. Analysis shows an essential influence of both time-dependent heat-generation coefficient and contact heat transfer coefficient on the partition of the friction heat between the layers. The solution can be used for simulating temperature fields in sliding components with account of this influence.

1. Introduction

Thermal friction problem is considered to be one of the central problems in tribology due to the fact that thermal effects manifest in various forms and often can not be ignored. Accurate simulation of frictional processes in a tribosystem requires the knowledge of the temperature fields in its sliding components. These temperature fields can be determined by numerical methods. Despite the fact that such methods have become common practice, the analytical approach is preferable in many cases. Analytical expressions for the temperature fields enable parametric analysis, investigation of asymptotic behavior and special cases, or testing of numerical algorithms.

Thermal friction problem is usually formulated in the form of an initial-boundary-value problem of non-stationary heat conduction in two coupled bodies with a heat source at their interface. The contact conditions are specified at the interface to describe a certain relation between the spatial derivatives of the temperatures T_1 and T_2 of the bodies, i.e. the heat fluxes into the bodies, and the specific power q of heat generation. There are several basic types of the contact conditions, which are reviewed in [1].

Blok [2] partitioned the friction heat between the sliding bodies by introducing the heat-partition coefficient α_f, so that the contact conditions have the form

$$\lambda_1 \frac{\partial T_1}{\partial \vec{n}}\bigg|_S = \alpha_f q, \quad -\lambda_2 \frac{\partial T_2}{\partial \vec{n}}\bigg|_S = (1-\alpha_f)q, \tag{1}$$

where S is the interface region, $\vec{n}$ is the unit normal vector at S directed from the first to the second body, λ_1 and λ_2 are the thermal conductivity coefficients of the bodies.

Ling [3] formulated the conditions of the perfect thermal contact which imply the energy balance and temperature continuity in the microscopic regions of contact of roughness asperities. The perfect thermal contact conditions are also often specified at the macroscopic interface, that is,

$$\lambda_1 \frac{\partial T_1}{\partial \vec{n}}\bigg|_S - \lambda_2 \frac{\partial T_2}{\partial \vec{n}}\bigg|_S = q, \quad T_1|_S = T_2|_S. \tag{2}$$

Podstrigach [4] considered a thermal interaction of two bodies through a thin intermediate layer. He proposed the conditions of imperfect thermal contact between the bodies which describe the heat conduction in the intermediate layer with the contact heat transfer coefficient γ. In the presence of a heat source at the interface, these conditions take the form

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$$\lambda_1 \frac{\partial T_1}{\partial \vec{n}}\bigg|_S = \frac{q}{2} - \gamma(T_1 - T_2)|_S,$$
$$-\lambda_2 \frac{\partial T_2}{\partial \vec{n}}\bigg|_S = \frac{q}{2} + \gamma(T_1 - T_2)|_S. \quad (3)$$

Independently, Barber [5,6] and Protasov [7] introduced another type of imperfect thermal contact conditions, which can be presented in our notations as

$$\lambda_1 \frac{\partial T_1}{\partial \vec{n}}\bigg|_S = \alpha q - \gamma(T_1 - T_2)|_S,$$
$$-\lambda_2 \frac{\partial T_2}{\partial \vec{n}}\bigg|_S = (1-\alpha) q + \gamma(T_1 - T_2)|_S, \quad (4)$$

where α is the heat-generation coefficient [7]. It is noteworthy that Barber's reasoning [5] was based on heat conduction theory. He assumed that the heat flux passing in either of the sliding bodies consists of two components: the first one is due to the frictional heating, while the second is caused by the temperature difference of the bodies. The coefficient α is determined through the microscopic thermal resistances of the rough surfaces of the bodies. Protasov [7], on the other side, investigated the friction heat generation considering adhesion-deformational interactions of roughness asperities and based his conclusion on the principles of thermodynamics. He introduced α as the fraction of the friction energy which is generated at the surface (adhesive mechanism) and in the subsurface layer (deformational mechanism) of the first sliding body.

There is a principal difference between α_f and α: the former means the partition of the friction heat, whereas the latter specifies the partition of the heat-generation power (q) between the sliding bodies. When using any of the conditions (2), (3), or (4), α_f is a priori unknown.

It should be mentioned that the contact conditions (4) are a generalization of the contact conditions considered above, so that (4) would degenerate into (1) at $\gamma = 0$, into (2) at $\gamma \to \infty$, and into (3) at $\alpha = 1/2$.

If the friction conditions, such as the sliding velocity and contact pressure, vary with time, this would result in a change of q. They have also effects on the coefficients α and γ. It is known from literature (see, for instance, [8]) that γ generally increases with the contact pressure. According to the theoretical study [9], both α and γ depend on the sliding velocity. Thus, the quantities q, α, γ should be considered as variables dependent on time t. By this means, the contact conditions (4) are transformed into

$$\lambda_1 \frac{\partial T_1}{\partial \vec{n}}\bigg|_S = \alpha(t) q(t) - \gamma(t)(T_1 - T_2)|_S,$$
$$-\lambda_2 \frac{\partial T_2}{\partial \vec{n}}\bigg|_S = (1-\alpha(t)) q(t) + \gamma(t)(T_1 - T_2)|_S. \quad (5)$$

A number of analytical studies on temperatures in sliding components have been conducted using the Laplace transform, Fourier series, or other techniques. The existing mathematical techniques allow to derive analytical solutions of one-dimensional heat conduction problems for sliding bodies represented in the form of semispaces or layers. Classical solutions for the semispaces interacting due to the contact conditions (1) or (2) can be found in [10]. Temperature expressions for the elements of the pairs semispace–semispace, semispace–layer and layer–layer were derived for the contact conditions (3) [11–14], and the contact conditions (4) [15–17]. At the same time, a literature review reveals that analytical solutions of heat conduction problems with the time-dependent conditions (5) are unknown.

The aim of this study is to provide an analytical solution of the initial-boundary-value problem of non-stationary heat conduction in two layers coupled through the contact conditions (5). For this purpose, an original integral transform is developed to map the differential operator under the given specific contact conditions.

2. Problem statement

We consider non-stationary heat conduction in two layers with thicknesses h_1 and h_2 which move relative to each other, so that there is friction between them. The friction leads to a heat release with a specific power given as a continuous function $q(t) \geqslant 0$. We assume that the thermal contact between the layers follows the conditions (5). At the free surfaces of the layers we specify convective heat transfer with coefficients α_1 and α_2. At $t = 0$ the temperatures of the layers are equal to some ambient temperature $T_0 \neq 0$. Under the given assumptions, the temperatures $T_1(x,t)$ and $T_2(x,t)$ in the layers change with time t along the direction x perpendicular to the sliding interface. Fig. 1 shows a schematic of the problem. The thermal conductivity and diffusivity coefficients of the layers are denoted by λ_1, λ_2 and a_1, a_2, respectively.

The dimensionless formulation of the problem described incorporates the heat conduction equations

$$\frac{\partial \Theta_1(\xi,\tau)}{\partial \tau} = \frac{\partial^2 \Theta_1(\xi,\tau)}{\partial \xi^2}, \quad -1 < \xi < 0, \quad \tau > 0,$$
$$\frac{\partial \Theta_2(\xi,\tau)}{\partial \tau} = \chi \frac{\partial^2 \Theta_2(\xi,\tau)}{\partial \xi^2}, \quad 0 < \xi < H, \quad \tau > 0 \quad (6)$$

with zero initial conditions

$$\Theta_1(\xi,\tau)|_{\tau=0} = 0 = \Theta_2(\xi,\tau)|_{\tau=0}, \quad (7)$$

the contact conditions

$$\frac{\partial \Theta_1(\xi,\tau)}{\partial \xi}\bigg|_{\xi=0} = \alpha(\tau) Q(\tau) - B(\tau)(\Theta_1(\xi,\tau) - \Theta_2(\xi,\tau))|_{\xi=0},$$
$$-\frac{1}{\Lambda}\frac{\partial \Theta_2(\xi,\tau)}{\partial \xi}\bigg|_{\xi=0} = (1-\alpha(\tau)) Q(\tau) + B(\tau)(\Theta_1(\xi,\tau) - \Theta_2(\xi,\tau))|_{\xi=0} \quad (8)$$

and the boundary conditions

$$\frac{\partial \Theta_1(\xi,\tau)}{\partial \xi}\bigg|_{\xi=-1} = \mathrm{Bi}\,\Theta_1(\xi,\tau)|_{\xi=-1},$$
$$-\frac{1}{\Lambda}\frac{\partial \Theta_2(\xi,\tau)}{\partial \xi}\bigg|_{\xi=H} = \Upsilon \mathrm{Bi}\,\Theta_2(\xi,\tau)|_{\xi=H}. \quad (9)$$

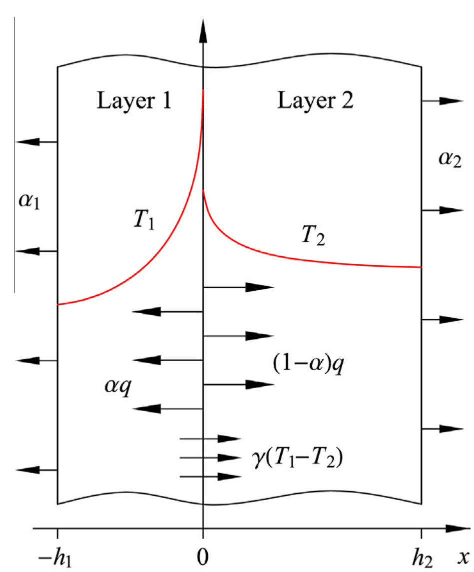

Fig. 1. Problem schematic.