International Journal of Heat and Mass Transfer 96 (2016) 164–170

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analytical solutions of the equations for the transient temperature field in the three-fluid parallel-channel heat exchanger with three thermal communications

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ARTICLE INFO

Article history:
Received 8 October 2015
Received in revised form 1 January 2016
Accepted 1 January 2016

Keywords:
Three-fluid parallel-channel heat exchanger
Co-flow
Counter-flow
Analytical solution
Transient case

ABSTRACT

The partial differential equations for the general case of the three-fluid parallel-channel heat exchanger are solved analytically by formulating them into boundary control systems. Both the co- and the counter-flow cases with three thermal communications between the channels are considered. Based on the analytical solution formula for the transient case, the steady state and the time to reach the steady state are studied. Illustrating examples and numerical simulations are given.

1. Introduction

As important components of numerous thermal systems, *heat exchangers* are widely used in a variety of industrial processes and engineering experiments to realize the transfer of heat between hot and cold fluids or gases [10], for instance in: refrigeration, air conditioning, power plants, chemical plants, petrochemical plants, petroleum refineries, natural gas processing, and sewage treatment. Among which, the *three-fluid parallel-flow* and the *three-fluid counter-flow heat exchangers* or the mixed ones, are popular and commonly utilized in many places, such as: helium-air separation units, ammonia gas synthesis, hydrogen liquefaction processes, and purification systems.

In this paper, we are concerned with the following equations for the three-fluid parallel-channel heat exchanger with three thermal communications

$$\begin{cases} \frac{\partial T_1}{\partial t}(x,t)+s_1 v_1 \frac{\partial T_1}{\partial x}(x,t)=b_{12}[T_2(x,t)-T_1(x,t)]+b_{13}[T_3(x,t)-T_1(x,t)], \\ \frac{\partial T_2}{\partial t}(x,t)+s_2 v_2 \frac{\partial T_2}{\partial x}(x,t)=b_{21}[T_1(x,t)-T_2(x,t)]+b_{23}[T_3(x,t)-T_2(x,t)], \\ \frac{\partial T_3}{\partial t}(x,t)+s_3 v_3 \frac{\partial T_3}{\partial x}(x,t)=b_{31}[T_1(x,t)-T_3(x,t)]+b_{32}[T_2(x,t)-T_3(x,t)]. \end{cases} \quad (1)$$

Here, $T_i(x,t)$ denotes the temperature of the fluid in the channel i at time t and at position $0 \leqslant x \leqslant l$, the letter l represents the length of the heat exchanger, $s_i = \pm 1$ indicates the direction of the fluid i, the positive constants b_{ij} are of dimension Hz,

$$b_{ij} = \frac{k_{ij} P_{ij}}{\rho_i c_{pi} A_i} \quad (2)$$

where $k_{ij} = k_{ji}, P_{ij} = P_{ji}$ $(i,j = 1,2,3; i \neq j)$. Note that strictly speaking, each b_{ij} depends on other physical coefficients that depend on temperature and hence on position and time. The constant assumption is an approximation which makes the problem easy to deal with.

The above type of heat exchanger equations has been studied in [1, Section 2] where it was assumed that the heat transfer coefficients and the physical properties of the fluids are constant. The fluid temperature and mass flow rate in each channel were supposed to be uniform on the cross section perpendicular to the stream direction. In addition, unlike [9], the axial heat diffusion in the walls and the heat capacity of the walls were not taken into account.

The transient temperature fields in multi-channel heat exchangers and related topics have received much attention in recent years [1–3,5,7–9,12,13]. Different methods have been adopted and different topics have been concerned. Explicit solutions of the equations for the three-fluid heat exchanger at steady states have been derived for all fluid flow arrangements [14], see

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.01.001
0017-9310/

Nomenclature

symbol	*physical meaning and dimension*	$S_i(t)$	the i-th term of the Dyson–Philips series
A_i	cross section area of channel i, m^2	$\mathbb{S}_l(t)$	the left-shift semigroup on $L^2(0,l)$
b_{ij}	quantities given by (2), Hz	$\mathbb{S}_r(t)$	the right-shift semigroup on $L^2(0,l)$
c_{pi}	specific heat of fluid i at constant pressure, J/(kg K)	t	time, s
$H^1(0,l)$	the first order Sobolev function space	$T_i(x,t)$	the transient temperature profiles, °C
k_{ij}	overall heat transfer coefficient between channels i and j, W/(m K)	$\mathbb{T}(t)$	the system operator semigroup
l	common length of the channels, m	$u_i(t)$	the system boundary input into channel i
$L^2(0,l)$	the function space of squarely integrable functions	v_i	velocity of the fluid in channel i, m/s
P_{ij}	common perimeter of channels i and j, m	x	Cartesian coordinate, m
s_i	flow direction of fluid i, $s_i = 1$ means positive x-direction and $s_i = -1$ means negative x-direction	Z	$Z = L^2(0,l) \times L^2(0,l) \times L^2(0,l)$, a product space
		$\phi_i(x)$	initial temperature field of the fluid i
		ρ_i	density of the fluid i, $\mathrm{kg/m^3}$

Fig. 1 therein for all the possible arrangements. Under some additional conditions, an analytical solution was obtained in [1] applying the Laplace transform method. However, to the knowledge of the authors, what obtained in the literature are mostly either numerical solutions or the analytical stationary field solution. In order to obtain stationary initial conditions and stationary final state for (1), its stationary case is solved with relevant boundary conditions. Well suitable for this purpose is the effective *shooting method* [6].

Recently, we solved analytically the three-fluid parallel-flow heat exchanger equation with two thermal communications for the case $s_1 = s_2 = s_3 = 1$ by formulating it into a boundary control system, see [2] for the details. In the present paper, this method is adopted again to study the general case of equations for the three-fluid parallel-channel heat exchanger with three thermal communications.

The rest of this paper is organized as follows. In Section 2, we solve (1) analytically. Based on the analytical solutions, the steady state and the steady state time are then derived which follows from the nilpotence of the underlying system semigroup. In Section 3, illustrating examples and numerical simulations are given. Section 4 is devoted to conclusions.

2. Analytical solutions

In this section, we solve (1) analytically. As mentioned before, the state space approach is adopted and we work in the real state space

$$Z := L^2(0,l) \times L^2(0,l) \times L^2(0,l)$$

where $L^2(0,l)$ stands for the space of squarely integrable functions on $(0,l)$. The standing assumptions on the initial temperature fields $T_i(x,0) = \phi_i(x)$ and the boundary inputs $u_i(t)$ are

$$\phi_i \in H^1(0,l), \qquad u_i(\cdot) \in C^2[0,\infty), \quad i = 1,2,3 \tag{3}$$

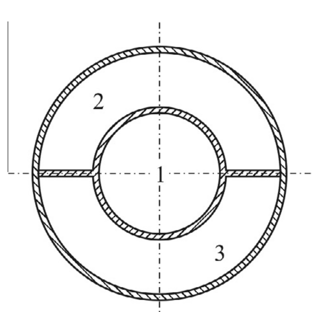

Fig. 1. Cross section of the heat exchanger with three thermal connections.

where $H^1(0,l)$ is the first order Sobolev function space (differentiable functions in the sense of *distribution*, equivalently, *absolutely continuous* and hence differentiable almost everywhere with derivative in $L^2(0,l)$; we refer to any a standard textbook on functional analysis, for instance [11, Section 7.2] for more information).

All the following possible fluid flow arrangements will be considered (note that Case 1 with two thermal communication has been studied in [2]).

Case 1: $s_1 = s_2 = s_3 = 1$.
Case 2: $s_1 = s_3 = 1, \quad s_2 = -1$.
Case 3: $s_1 = 1, \quad s_2 = s_3 = -1$.
Case 4: $s_1 = s_2 = 1, \quad s_3 = -1$.

The boundary conditions are possible combinations of $T_i(0,t) = u_i(t)$ and $T_j(l,t) = u_j(t)$. The concrete form depends on the fluid flow arrangement. For example, the boundary conditions corresponding to Case 2 are

$$T_1(0,t) = u_1(t), \qquad T_2(l,t) = u_2(t), \qquad T_3(0,t) = u_3(t). \tag{4}$$

As a state space method, the boundary control system method [4, Section 3.3] is classical and effective in solving linear partial differential equations with boundary inputs. In the following, we solve (1) and (3) case by case. Since the discussions are in parallel to those in [2], here we only state the results. Let $\mathbb{S}_r(t)$ and $\mathbb{S}_l(t)$ be the right-shift and left-shift semigroups on $L^2(0,l)$ given by

$$(\mathbb{S}_r(t)f)(x) = \begin{cases} f(x-t), & x \geqslant t, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

$$(\mathbb{S}_l(t)f)(x) = \begin{cases} f(x+t), & x+t \leqslant l, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

2.1. Analytical solution for Case 1

For Case 1, the boundary conditions are

$$T_1(0,t) = u_1(t), \qquad T_2(0,t) = u_2(t), \qquad T_3(0,t) = u_3(t), \tag{7}$$

if

$$\phi_1(0) = u_1(0), \qquad \phi_2(0) = u_2(0), \qquad \phi_3(0) = u_3(0), \tag{8}$$

then (1) with $s_1 = s_2 = s_3 = 1$ subject to (3) and (7) has the solution

$$\begin{aligned} \begin{bmatrix} T_1(x,t) \\ T_2(x,t) \\ T_3(x,t) \end{bmatrix} &= \begin{bmatrix} (x+1)u_1(t) \\ (x+1)u_2(t) \\ (x+1)u_3(t) \end{bmatrix} + \mathbb{T}(t) \begin{bmatrix} \phi_1(x) - (x+1)u_1(0) \\ \phi_2(x) - (x+1)u_2(0) \\ \phi_3(x) - (x+1)u_3(0) \end{bmatrix} \\ &+ \int_0^t \mathbb{T}(t-s) \begin{bmatrix} U_{11}(x,s) \\ U_{12}(x,s) \\ U_{13}(x,s) \end{bmatrix} \mathrm{d}s. \end{aligned} \tag{9}$$