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Entrance length effects on Graetz number scaling in laminar duct flows with periodic obstructions: Transport number correlations for spacer-filled membrane channel flows

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ABSTRACT

Self-similarity and scaling laws are powerful tools in engineering and thus useful for the design of apparatus. This self-similarity is well understood for the heat and mass transfer in laminar empty channel flows, including the fully developed region as well as inlet length effects in the developing region (Graetz problem). In this study, we examine the validity of the scaling behavior arising from the Graetz solution for channel flows disturbed by periodic obstructions. Simulation results show that entrance length effects and scaling laws do not change due to the presence of obstructions if the flow field remains steady in time and the dimensionless inlet length is given by $X_T/D_h \approx C_{inl.} \cdot \mathrm{Re} \cdot \mathrm{Pr}$, where $C_{inl.} \approx 0.01$ for the local and $C_{inl.} \approx 0.03$ for the average Nusselt number. The Nusselt number in the inlet region for an internal flow scales by $\mathrm{Nu} = (\mathrm{Re} \cdot \mathrm{Pr})^{1/3}$, similar to the empty channel flow (Shah and London, 1978). If the analogy between heat and mass transfer holds, same conclusions and relations are valid for the Sherwood number, $\mathrm{Sh} \propto (\mathrm{Re} \cdot \mathrm{Sc})^{1/3}$, where Sc denotes the Schmidt number. In the fully developed region, the Nusselt number depends slightly on the Reynolds and Prandtl numbers owing to the loss in self-similarity of the velocity field (contrary to the empty channel flow). The limit of the classical self-similarity is the onset of temporal oscillations (instability) in the flow field. Beyond this limit, the length of the thermal entrance region is strongly reduced. Furthermore, a strong dependency of the Nusselt number in the fully developed region on the Prandtl number is found.

1. Introduction

Heat and mass transport in laminar forced convection channel flows occurs in many processes and thus has been widely investigated theoretically, experimentally, and numerically. Many literature reviews have been written (for instance the book by Shah and London reviewing analytical solutions for laminar channel flows [1]) and the fundamentals of laminar convective flows in circular ducts are addressed in standard heat transfer textbooks [2–4]. Thus, approximate solutions of the velocity and temperature profiles in the developing and fully developed regions are well known. The complexity of the problem increases for non-circular cross-sections [5] or flows inside concentric annuli [6,7], non-Newtonian fluids or temperature dependent fluid properties [8], buoyancy forces (mixed convection), and more complex thermal boundary conditions (temporally or spatially varying) at the channel walls [9,6,7] or the channel inlet [10]. For many laminar flow configurations, analytical solutions of the Graetz problem are given in literature. Further, also for turbulent flow conditions analytical solutions are possible, assuming a turbulent velocity profile together with a distribution of the eddy diffusivity [11,12]. An increasing interest in laminar flow regimes is also driven by the investigation of microchannels, such that Graetz number scaling behavior can be found in many correlations developed for heat and mass transfer in single phase microchannel flows [13]. Even the thermal development of forced convection flows through porous materials can be described by the Graetz number [14–16]. Thereby, Nield et al. [14] showed that the developing Nusselt number varies only slightly with the Darcy number but quiet drastically with the Peclet number, owing to the influence of axial conduction.

The motivation of this study is driven by the mass transfer in laminar forced convection channel flow that is typical for membrane technologies such as electrodialysis, reverse osmosis, ultrafiltration [17], and nano-filtration [18]. The channels of these modules are filled with mesh spacers, which separate the

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Nomenclature

c_p	specific heat capacity [J/kg K]
D_h	hydraulic diameter [m]
D_s	spacer diameter [m]
$\mathcal{D}$	mass diffusivity [m^2/s]
h	heat transfer coefficient [W/m^2 K]
H	channel height [m]
k	thermal conductivity [W/m K]
k_c	transfer coefficient [m/s]
L_s	spacer separation distance [m]
p	pressure [kg/m s^2]
q	specific heat flux [W/m^2]
t	time [s]
T	temperature [K]
u	velocity [m/s]
x	streamwise coordinate [m]
y	crosswise coordinate [m]
α	thermal diffusivity [m^2/s]
δ	hydrodynamic b.l. thickness [m]
δ_T	thermal b.l. thickness [m]
ρ	density [kg/m^3]
ν	kinematic viscosity [m^2/s]

Dimensionless numbers

$\mathrm{Nu} = \frac{hD_h}{k}$ Nusselt number
$\mathrm{Pe} = \mathrm{Re} \cdot \mathrm{Pr}$ Peclet number
$\mathrm{Pr} = \frac{\nu}{\alpha}$ Prandtl number
$\mathrm{Re} = \frac{u_0 D_h}{\nu}$ Reynolds number
$\mathcal{R}e = \frac{u_0 D_s}{\nu}$ Reynolds number
$\mathrm{Sc} = \frac{\nu}{\mathcal{D}}$ Schmidt number
$\mathrm{Sh} = \frac{k_c D_h}{\mathcal{D}}$ Sherwood number

Subscripts

0	reference value
q	constant heat flux b.c.
T	constant temperature b.c.
W	Wall
–	spatially averaged quantity

membrane walls of feed flow and permeate thus creating a flow passage. The spacers usually consist of crossed, almost cylindrical filaments which can be woven or non-woven. Besides their primary task of separating the membrane walls, they should also promote mass transfer, reduce fouling and reduce the phenomenon of concentration polarization near the membrane walls. The enhancement of transport (mass or heat) in internal flows is a general topic and not only addressed in membrane channel flows. Thus, various methods have been proposed in literature, ranging from twisted tapes [19], internal fins, wire coil, mesh or brush inserts, up to displaced mixing devices such as spaced disks, spaced streamline shapes, flow twisting device [20] or louvered stripes, for a review of heat transfer enhancing methods see [21–24]. A second characteristic of the mass transfer in membranes is the high Schmidt number, in the order of thousand for system feed waters [25]. Owing to the high Schmidt number, this transport is dominated by convection.

In recent years, many theoretical, experimental and numerical studies have been conducted with the aim of understanding the flow phenomena involved in laminar forced convection in spacer-filled ducts, as reviewed in Karabelas et al. [18]. Further, optimization of the spacer configuration has been addressed, whereby the desired enhancement in mass transfer is directly coupled to an undesired increase in pressure drop. Despite the large number of studies in this field, little attention is drawn to entrance length effects. It is commonly assumed that "the effect is less important when spacers are involved as they introduce a breakup of the boundary layer on the spacer dimension ..." [26].

However, it is well known from laminar plane channel flows that the dimensionless transport number (Nusselt or Sherwood number) approaches a constant value, after some entry length in which the boundary grows, that is independent of Reynolds and Prandtl numbers. Thus, the question arises: due to which mechanisms and at which critical obstacle or spacer dimensions (size and separation distance) does the scalar transport in a spacer-filled channel flow deviate from the scalar transport in a plane channel flow?

The remainder of this paper is structured as follows. Section 2 provides a general description of the hydrodynamic case studied, introducing the governing equations and relevant dimensionless numbers. Section 3 gives a short overview of the numerical methods used and presents a grid dependency analysis showing the robustness of the results obtained. Before discussing the results obtained for the spacer-filled channel, scaling laws in laminar developing plane channel flows are briefly recalled in Section 4. Although this section presents well-known text book information, its content is essential for understanding more complex flow configurations. The main contribution of this paper is in Section 5, which provides simulation results and a general understanding of the physical mechanisms involved in the developing channel flow with periodic obstacles. The results section is followed by a short discussion of the results with respect to membrane channel flows in Section 6. Further, the new insights provided by this study are used to discuss the results obtained in earlier experimental and numerical studies and to provide guidelines for further investigations. Finally, Section 7 summarizes the findings.

2. Problem statement, dimensionless parameters, and governing equations

The configuration of the channel flow investigated in this study is schematically shown in Fig. 1. An infinite width of the channel is assumed that allows for two-dimensional simulations. This assumption neglects crosswise and three-dimensional perturbations, limiting the validity of the simulation results to low Reynolds number cases. Note that a transition to three-dimensional wake regime is found to occur for the unconfined flow across a cylinder at $\mathcal{R}e = 180 - 194$ (defined later), depending on experimental conditions [27]. The length of the entire duct is chosen to be sufficiently long in order to investigate inlet length effects, but also to allow for fully resolved numerical simulations to be conducted in a reasonable time. In this study, most of the results for the inlet behavior are computed using a duct length of 200 successive spacers. The number of possible flow configurations is reduced by limiting the investigation to obstacles located in the center of the duct. However, many of the conclusions obtained for centered obstacles are also valid for off-center or wall-aligned obstacles.

Using the Einstein summation rule the governing equations for a time-dependent flow of an incompressible fluid with constant properties are

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (1)$$