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Velocity and thermal slip effects on peristaltic motion of Walters-B fluid

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ABSTRACT

The aim of present attempt is to address the peristaltic transport of Walters-B fluid in a compliant wall channel. Problem formulation and analysis is presented in the presence of both velocity and thermal slip condition. The solution expressions of stream function, velocity, temperature and heat transfer coefficient are derived. Effects of various involved key parameters on the physical quantities are sketched and discussed.

1. Introduction

The transportation of many biological fluids is carried out with the help of naturally inherited mechanism inside the biological systems which is called peristalsis. It is nature's way of moving the content within hollow muscular structures by successive contraction of their muscular fibers. This principle is responsible for transport of biological fluids such as urine in the ureter, chyme in the gastrointestinal tract, semen in the vas deferens, ovum in the fallopian tube, lymph transport in the lymphatic vessels, blood pumps in the heart lung machine etc. In plant physiology, the peristalsis is involved in phloem translocation by driving a sucrose solution along tubules by peristaltic contractions. The corrosive and noxious fluids can also be transported by peristalsis. Such flows in presence of heat transfer also have great value. This process is useful for the analysis of tissues, oxygenation and dialysis. For instance, the blood flow through dilution technique can be monitored. The heat in this mechanism is either injected or induced locally and the thermal clearance is noted. The blood flow rate is estimated through information of initial thermal conditions and the thermal clearance rate. In the recent years, the application of heat (hyperthermia), radiation (laser therapy) and coldness (cryosurgery) has attracted the attention of the researchers in thermal modeling for the destruction of undesirable tissues in cancer therapy. Non-Newtonian fluid mechanics has emerged as one of most important research areas of modern applied mathematics. The elastic characteristics of many biological and industrial fluids cannot be ignored because most of them are the suspensions of particles in the Newtonian liquid with short memory. Due to this reason, almost all biological and industrial fluids are non-Newtonian. Peristalsis due to non-Newtonian fluids has been given special attention by the researchers in recent years. For instance, Abd elmaboud and Mekheimer [1] discussed the peristaltic transport of second order fluid filling a porous space. Tripathi and Beg [2] analyzed the magnetohydrodynamic effects on the peristaltic motion of couple stress fluids. They observed that couple stress effects have significant role on the solutions. Numerical solutions for peristaltic motion of Carreau–Yasuda fluid in a curved channel have been provided by Hayat et al. [3]. Peristalsis of electrically conducting Carreau fluid through a tapered asymmetric channel was described by Kothandapani et al. [4]. Kothandapani and Prakash [5] investigated the combined influence of magnetic field and thermal radiation on the peristaltic flow of Williamson fluid with nanoparticles. Rotational effects on the peristaltic flow of fourth grade fluid under the influence of magnetic field were reported by Abd-Alla et al. [6]. They found that magnetic field and rotational effects tend to oppose the fluid motion. Mustafa et al. [7] employed Keller-box method to study mixed convection in the peristaltic transport of fourth-grade fluid. They observed that buoyancy forces stemming due to the temperature and concentration gradients strongly alter the flow fields. Further recent studies in this direction can be found in Refs. [8–13].

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Flow pattern characterized by the slip boundary condition has special significance in many applications. In some situations, the fluid presents a loss of adhesion at the wetted wall which compels it to slide along the wall. The concept of wall slip is important in describing the macroscopic effects of certain molecular phenomena in the study of fluid–solid interaction problems. Consideration of no-slip condition seems unrealistic for many non-Newtonian flows because they exhibit macroscopic wall slip. Particularly, no-slip condition is inadequate for rough surfaces and in micro-electro-mechanical-systems (MEMS). The fluids exhibiting boundary slip finds applications in technology such as in the polishing of artificial heart. Various researchers studied the peristaltic flow subject to slip boundary condition. For example, Chu and Fang [14] discussed the peristaltic flow considering slip condition. Ali et al. [15] investigated the slip effects on peristaltic flow of MHD fluid with variable viscosity. Chaube et al. [16] addressed the slip effects on peristaltic flow of micropolar fluid. Hayat and Hina [17] discussed the slip and heat and mass transfer effects on peristaltic flow of Williamson fluid in a non-uniform channel. Hina et al. [18] provided numerical and analytic solutions for peristaltic motion of Johnson–Segalman fluid considering slip conditions. In another paper, Slip effects on the flow of pseudoplastic fluid with peristalsis were examined by Hina et al. [19]. Recently, Ellahi et al. [20] explored the partial slip effects on the peristaltic motion of Jeffrey fluid through a rectangular duct.

Consideration of wall properties in peristalsis is of special value in study of blood flow in arteries and veins, urine flow in the urethras and air flow in the lungs. Peristaltic motion in a complaint wall channel has also been investigated by some researchers. Radhakrishnamacharya and Srinivasulu [21] analyzed the influence of wall properties on peristaltic motion of Newtonian fluid with heat transfer. Peristaltic motion of micropolar fluid in circular cylindrical tubes with wall properties is discussed by Muthu et al. [22]. Hayat et al. [23,24] examined the MHD peristaltic flow of Jeffery and Johnson–Segalman fluids with compliant walls. Srinivas and Kothandapani [25] analyzed the heat and mass transfer effects on MHD peristaltic flow of Newtonian fluid in a porous channel with compliant walls. Riaz et al. [26] investigated the peristaltic motion of Prandtl fluid in rectangular duct with wall properties. Recently, peristaltic flow of Burgers' fluid in a complaint walls channel was investigated by Javed et al. [27]. Peristaltic flow with complaint walls and Hall current was studied by Gad [28]. Very recently, the combined influence of heat and mass transfer on the peristaltic motion of pseudoplastic fluid with wall properties was analytically explored by Hina et al. [29]. Very recently, slip effects on the peristaltic flow of Eyring–Powell fluid with wall properties were examined by Hina [30].

Amongst the many suggested models, Walters [31] has developed a physically accurate mathematical model for the rheological equation of state of a viscoelastic fluid with short memory. This model has been shown to capture the characteristics of actual viscoelastic polymer solutions, hydrocarbons, paints and other chemical engineering fluids. The Walters-B model generates highly nonlinear flow equations which have order higher than that of the Navier–Stokes equations. It also incorporates elastic properties of the fluid which are important in extensional behavior of polymers. To the best of our knowledge, peristalsis of Walters'B fluid with wall properties has never been addressed previously. Thus present work is undertaken to fill this void by incorporating velocity slip and temperature jump conditions. A regular perturbation technique is employed to solve the present problem. The wave number in the perturbation solution is taken small. The expressions for the stream function, temperature distribution, velocity and heat transfer coefficient are calculated. Influence of emerging parameters is shown and discussed on the velocity, temperature distribution and heat transfer coefficient.

2. Mathematical model

For an incompressible fluid, the balance of mass and momentum are given by

$$\operatorname{div} V = 0, \tag{1}$$

$$\rho \frac{dV}{dt} = \operatorname{div}\mathbf{S} + \rho\mathbf{f}, \tag{2}$$

where ρ is the density, V is the velocity vector, $\mathbf{S}$ is the Cauchy stress tensor, $\mathbf{f}$ represents the specific body force, and d/dt represents the material time derivative. The constitutive equations for Walter's B fluid are

$$\mathbf{S} = -P\mathbf{I} + \varsigma, \tag{3}$$

$$\varsigma = 2\eta_0\mathbf{e} - 2k_0\frac{\bar{\delta}\mathbf{e}}{\bar{\delta}t}, \tag{4}$$

$$\mathbf{e} = \nabla V + (\nabla V)^T, \tag{5}$$

$$\frac{\bar{\delta}\mathbf{e}}{\bar{\delta}t} = \frac{\partial \mathbf{e}}{\partial t} + V.\nabla\mathbf{e} - \mathbf{e}\nabla V - (\nabla V)^T\mathbf{e} \tag{6}$$

in which $-P\mathbf{I}$ is the spherical part of the stress due to constraint of incompressibility, ς is the extra stress tensor, η_0 is the coefficient of viscosity, $\mathbf{e}$ is the rate of strain tensor and $\bar{\delta}/\bar{\delta}t$ denotes the convected differentiation of a tensor quantity in relation to the material motion.

We consider an incompressible Walters-B fluid in a channel of width $2d$. The channel walls are of compliant nature. The temperatures of the lower and upper walls of the channel are maintained at T_0 and T_1, respectively. The geometry of the wall is

$$y = \pm\eta = \pm\left[d + a\sin\left(\frac{2\pi}{\lambda}(x - ct)\right)\right],$$

where λ is the wavelength, c is the wave speed, a is the wave amplitude and x and y are the Cartesian coordinates with x measured in the direction of the wave propagation and y measured in the direction normal to the mean position of the channel walls.

Using Eqs. (1)–(6), the equations governing the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$\rho\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \tag{8}$$

$$\rho\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)v = -\frac{\partial p}{\partial y} + \frac{\partial S_{yx}}{\partial x} + \frac{\partial S_{yy}}{\partial y}, \tag{9}$$

$$S_{xx} = 4\eta_0\frac{\partial u}{\partial x} - 2K_0\left[2\frac{\partial^2 u}{\partial x\partial t} + 2\left(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial x\partial y}\right) - 4\left(\frac{\partial u}{\partial x}\right)^2 - 2\frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right], \tag{10}$$

$$S_{xy} = 2\eta_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - 2K_0\left[2\frac{\partial^2 u}{\partial y\partial t} + \frac{\partial^2 v}{\partial x\partial t} - 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{\partial v}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right], \tag{11}$$

$$S_{yy} = 4\eta_0\frac{\partial v}{\partial y} - 2K_0\left[2\frac{\partial^2 v}{\partial y\partial t} + 2\left(u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 v}{\partial y^2}\right) - 4\left(\frac{\partial v}{\partial y}\right)^2 - 2\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right], \tag{12}$$

$$\rho C_p\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)T = \xi\nabla^2 T + \Phi, \tag{13}$$

$$\Phi = S_{xx}\frac{\partial u}{\partial x} + S_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + S_{yy}\frac{\partial v}{\partial y}.$$