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A meshless method based on the method of fundamental solution for solving the steady-state heat conduction problems

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ABSTRACT

Motivated by the incompleteness of the method of fundamental solution (MFS) for the interior problem, we give another meshless method both theoretically and numerically for recovering the temperature and the heat flux based on the normal derivative of the fundamental solution in this paper. Although the problems under investigation are well-posed, we should note that the method presented here results in an ill-conditioned system and this is a feature of the numerical method employed in the present approach. The ill-posedness of this system is given by the potential function. In order to overcome the ill-posedness of the system, the Tikhonov regularization method, as well as Morozov's discrepancy principle for selecting an appropriate regularization parameter, are used to increase the stability of this method. Then three kinds of boundary value problems are presented to show the effectiveness of this method with some examples, whilst the comparisons with the MFS is presented. The numerical convergence and stability of this method are also analyzed.

1. Introduction

The MFS is an effective method for solving the direct/inverse problems governed by the partial differential equations. The idea is approximating the solution of the partial differential equations by a linear combination of the Green functions. Then the weighed coefficients can be determined by the boundary collocation method. Thus the MFS is also a meshless boundary collocation method, which belongs to Trefftz-like methods [1,2] and is applicable to BVPs in which a fundamental solution of the operator in the governing equation is known. Despite this restriction, the MFS has become very popular primarily because of the ease with which it can be implemented, particularly for the problems in complex geometries. The simple implementation of the MFS for the problems with complex boundaries makes it an ideal candidate for the problems in which the boundary is of major importance or requires special attention. Since then, it has been successfully applied to a large variety of physical problems, an account of which may be found in the survey papers [5,3,4]. However, there exist some heat conduction problems for which the simple application of the MFS is not sufficient to obtain an accurate numerical solution, e.g. problems related to domains containing a boundary singularity generated by the presence of a crack or a V-notch, and hence the standard MFS has to be modified/enriched.

Many researchers proposed to enrich their formulations of the MFS in the case of isotropic heat conduction problems, Alves and Leitao [6] introduced an enriched MFS to simulate a crack singularity. Particularly, Chen et al. [7] developed the singular boundary method (SBM) for avoiding of choosing a fictitious boundary. They assumed a test example to calculate the source weighting, and then used this source weighting to determine the value of diagonal term where the source and field points can coincide. However, for certain case, the SBM yields an inaccurate approximation. As a result, they provided an improved formulation of adding a constant term and a constraint [8,9]. They called this constraint moment condition. The SBM for solving the three-dimensional inverse heat conduction problems can be found in [10]. In order to give the completeness of the MFS in two-dimensions, Saavedra and Power [11,12] added a constant term in their formula. Although the constant in the MFS is sometimes recommended, it is rarely used in practice as the degenerate cases occurring rather rarely. Following Fichera's idea, Chen et al. [13] enriched the MFS by an added constant and a constraint. This enrichment is ensuring a unique solution of the problems considered. They also explained that this enrichment should be used when there is a degenerate scale. For a degenerate scale, we can refer to [14–18]. In [19,20], Sun and Ma have given a new view of this equation and derived it by using the invariance argument. The authors prove that the invariance property is the essence of the analytical solution. Marin [21] extended the modified MFS in [19,20] for Laplace–Beltrami's

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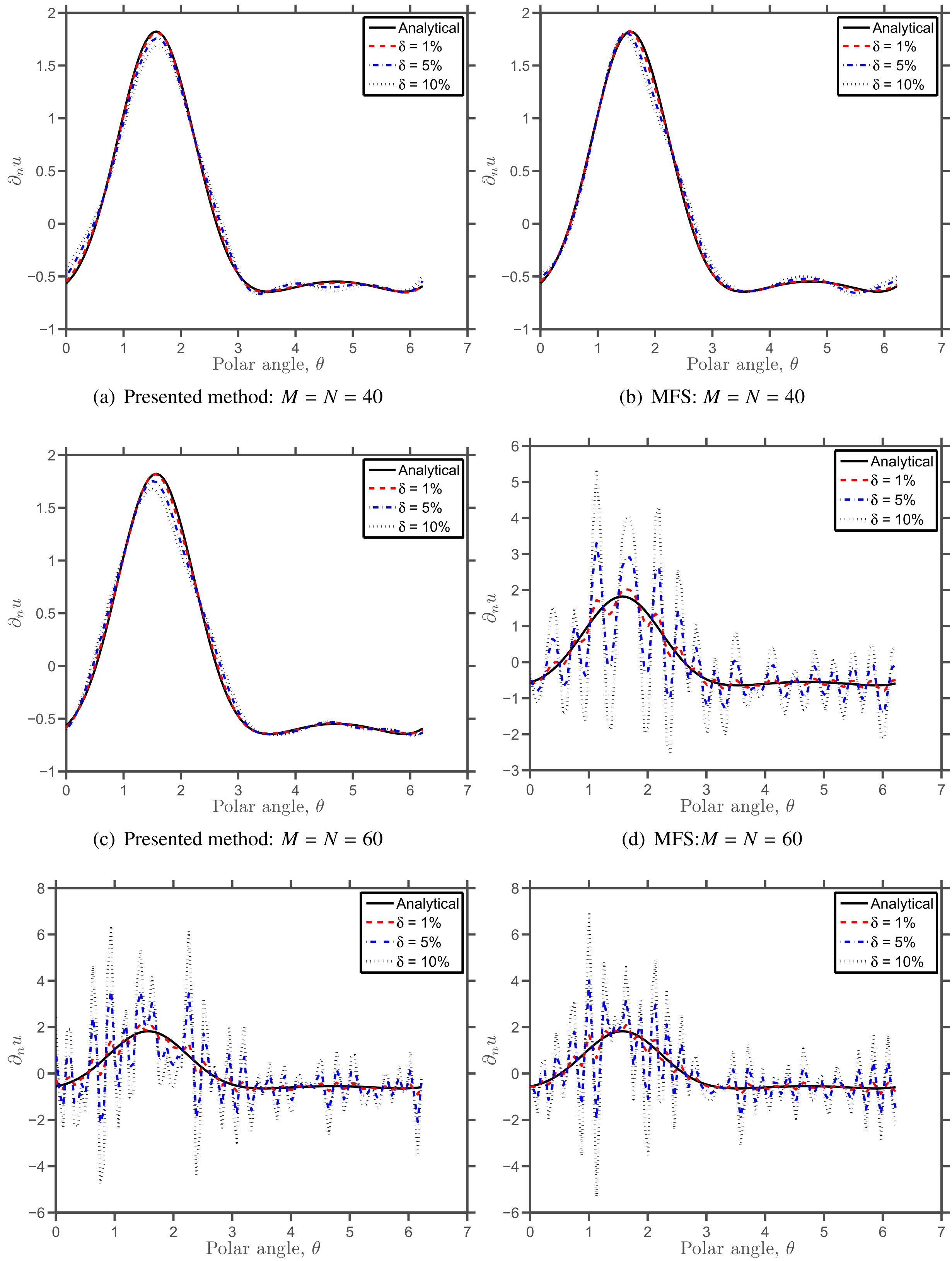
equation, i.e., in the case of two-dimensional anisotropic heat conduction problems.

The main goal of this paper is to give a simple and effective numerical method based on the MFS. The main idea is to approximate the solution by the derivative of the Green function, which is a solution of the Laplace equation

$$\Delta u = 0, \quad \text{in } \mathbb{R}^2 \setminus \{\boldsymbol{y}_j\}. \tag{1}$$



(a) Presented method: $M = N = 40$

(b) MFS: $M = N = 40$

(c) Presented method: $M = N = 60$

(d) MFS: $M = N = 60$

(e) Presented method: $M = N = 80$

(f) MFS: $M = N = 80$

Fig. 1. Example 1: the numerical solution of the normal heat flux on ∂D solving by the MFS and the presented method.