International Journal of Heat and Mass Transfer 96 (2016) 249–255

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Application of conjugate gradient method for estimation of the wall heat flux of a supersonic combustor

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ARTICLE INFO

Article history:
Received 5 June 2015
Received in revised form 12 January 2016
Accepted 12 January 2016
Available online 29 January 2016

Keywords:
Conjugate gradient method
Inverse heat conduction problem
Supersonic combustor
Wall heat flux

ABSTRACT

In this paper, conjugate gradient method (CGM) considering variations in material properties with temperature is developed to solve transient inverse heat conduction problems. With CGM, the measured outer wall temperatures of supersonic combustor are used as input data to recover the heat flux and temperature on the inner wall. Numerical test has been done to study the effect of temperature measurement noises on accuracy of the inversion result. It indicates that the inversion results are very close to the exact heat flux with a maximum error of less than 5% when the measurement noise is less than ±3 K. At the same time, a series of experiments are conducted on a Mach 3 supersonic combustor test facility with varied fuel injection conditions. The heat fluxes on the inner wall are recovered by measured outer wall temperatures via CGM. The inversion results of heat flux agree satisfactorily with the values measured by heat flux sensors and a maximum difference of less than 5% is found. The present comparison results prove the validity and accuracy of the CGM developed in the paper for estimation of the wall heat flux of supersonic combustor.

1. Introduction

Heat flux on the combustor inner wall is one of the most critical parameters for design and optimization of thermal protection of engine combustors. Both direct and indirect measurement techniques have been developed to measure the inner wall heat flux and temperature. Heat flux sensor, directly contacting the combustor flow, is the most commonly used apparatus [1,2]. However, its operation lifetime and reliability are limited due to combustor flow with high temperature, high speed and considerable fraction of oxygen. Additionally, because of its relatively large size especially with cooling structure, the spatial resolution of heat flux sensor is usually low and its response time is as large as a few seconds or more [3]. The indirect measurements include optical method, of which the inner wall temperature can be determined by infrared radiation of the heated wall surface [4,5]. However, intense emission from combustion flame and reflected light from other wall surfaces would cause significant errors in the measured wall temperature.

In this paper, time change of temperature on the outer wall of supersonic combustor is measured and used as the input data to determine the inner wall heat flux and temperature by conjugate gradient method (CGM) based on the principle of heat conduction through solid wall and calculus of variations. Oliveria and Orlande [6] evaluated surface heat flux of ablative material using CGM. Mohammadium et al. [7] studied accuracy and numerical stability of CGM when the measured temperatures (as the input data) had noises [8,9]. However, most of the previous conjugate gradient methods are implemented with the assumption that thermal properties of wall material such as thermal conductivity or specific heat are constant [7–9]. It is known that for gas turbine engine or ramjet/scramjet combustor, temperature across the combustor wall can vary from room temperature on the outer wall to more than 1000 K on the inner wall due to large heat flux with magnitudes of MW/m^2 [10,11]. The significant change in the wall temperature leads to large variations in thermal properties of wall material. Therefore, significant error in the recovered inner wall heat flux would exist if wall thermal parameters are still regard as constant in the inversion analysis. More importantly, most of the previous applications of CGM are for heat transfer analysis of reentry vehicles. To the authors' knowledge, application of CGM for thermal measurements of supersonic combustor has not been reported yet.

In the present study, CGM considering variations in material properties of the combustor wall is developed and the accuracy of the inversion results is examined when the input temperature has noises with different values. Using CGM, the measured outer-wall temperatures of a Mach 3.0 supersonic combustor via

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.01.036
0017-9310/

thermocouples or infrared measurements are used to determine the inner wall heat flux. The recovered inner wall heat fluxes are compared to the results obtained with commercial high-temperature heat flux sensors and good agreements between them are found. The present study proves feasibility and reliability of the present CGM for thermal measurements of supersonic combustor.

2. Description of the conjugate gradient method

A three-dimensional object of solid wall with heat loading is shown in Fig. 1. The initial temperature of the object is uniform and equals T_0. The heat flux $q_w(0,y,z,t)$ on the upper surface is a function of time as well as y and z distances. Four side-wall surfaces are assumed to be adiabatic since heat flux component in the direction vertical to the combustor wall is much larger than that in the direction perpendicular to the wall. The lower surface, representing the outer wall of combustor, is also considered to be adiabatic since heat transfer through the outer wall is dominated by natural convection and the heat flux value is negligible with comparison to the inner wall heat flux. With CGM, the measured temperature on the lower surface $T(L,y,z,t)$ is used as the input data to determine the heat flux on the upper surface.

The conjugate gradient method is used to solve unconstrained optimization problems such as inverse heat conduction problem described in Section 2.2. The purpose of CGM is to find a heat flux function on the upper surface that leads to a temperature distribution on the lower surface closest to the measured one. The problem has been transferred to a minimization problem of functional (Eq. (2) in Section 2.2). Assuming an initial guess of heat flux, the final function can be searched by Eq. (3a) in Section 2.3 in which the search direction can be obtained by gradient of functional as described in Section 2.5 and the search step size can be determined by solving sensitivity equation in Section 2.4.

2.1. Direct heat conduction problem

Formulation of direct heat conduction problem is given as follows:

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k(T)\frac{\partial T}{\partial z}\right] \quad \text{for} \quad 0 < x < L,\ 0 < y < M,\ 0 < z < N,\ t > 0 \tag{1a}$$

$$-k(T)\frac{\partial T}{\partial x} = q(y,z,t) \quad \text{for} \quad x = 0, t > 0 \tag{1b}$$

$$-k(T)\frac{\partial T}{\partial x} = 0 \quad \text{for} \quad x = L, t > 0 \tag{1c}$$

$$-k(T)\frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = 0, M, t > 0 \tag{1d}$$

$$-k(T)\frac{\partial T}{\partial z} = 0 \quad \text{for} \quad z = 0, N, t > 0 \tag{1e}$$

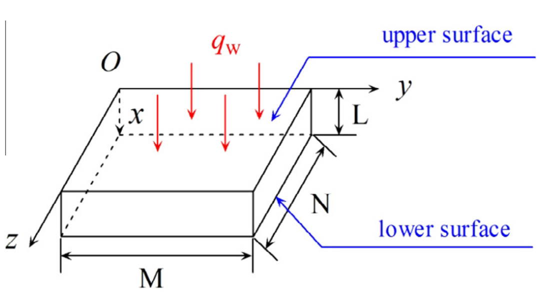

Fig. 1. Schematic diagram of the 3-dimensional wall with heating loading.

$$T = T_0 \quad \text{for} \quad 0 \leqslant x \leqslant L,\ 0 \leqslant y \leqslant M,\ 0 \leqslant z \leqslant N,\ t = 0 \tag{1f}$$

In the direct problem, heat flux on the upper surface is known and distribution of wall temperature at arbitrary time is calculated from Eq. (1) with 2nd-order finite difference scheme.

2.2. Inverse heat conduction problem

For the inverse problem, heat flux on the upper surface can be recovered by using the temperature on the lower surface. In view of high sampling rate of temperature measurement, the change of temperature can be regard as a continuous function of time. Therefore, the inverse problem can be solved by minimizing the following functional:

$$J[q(y,z,t)] = \int_0^{t_f} \sum_{i=1}^{i_m} [T(x_i,y_i,z_i,t) - Y(x_i,y_i,z_i,t)]^2 dt = \int_0^{t_f} \sum_{i=1}^{i_m} [T_i(t) - Y_i(t)]^2 dt \tag{2}$$

where, T and Y denote the calculated and the measured temperature on the lower surface respectively. The subscript "i" is ith moment during the measurement period.

2.3. Conjugate gradient method

The essential idea of CGM is that the minimization problem of the above functional (2) can be transformed into solutions of the direct problem of heat conduction, the sensitivity equation and the adjoint problem [11–15]. Hence, heat flux on the upper surface satisfying the temperature change on the lower surface can be obtained by the iterative process as described as follows:

$$q^{n+1}(y,z,t) = q^n(y,z,t) - \beta^n P^n(y,z,t) \quad \text{for} \quad n = 0, 1, 2, \ldots \tag{3a}$$

where β^n is the search step size from iteration n to $n + 1$. And P^n is the search direction given by the following conjugate equation:

$$P^n(y,z,t) = J'^n(y,z,t) + \gamma^n P^{n-1}(y,z,t) \tag{3b}$$

where, J'^n as called as the gradient direction, denotes derivative of the functional $J[q(y,z,t)]$ for time and for y and z distances and it is determined by solving the adjoint equation as described in Section 2.5. The conjugate coefficient γ^n is determined from:

$$\gamma^n = \frac{\int_0^{t_f} \sum_{i=0}^{i_m} [J'^n(y_i,z_i,t)]^2 dt}{\int_0^{t_f} \sum_{i=0}^{i_m} [J'^n(y_i,z_i,t)]^2 dt} \quad \text{with} \quad \gamma^0 = 0 \tag{3c}$$

To perform the iterative process given by Eq. (3a), the search step size β^n and search direction P^n are needed. To obtain the two parameters, a sensitivity problem and an adjoint problem are constructed in the following.

2.4. Sensitivity problem and search step size

Sensitivity problem can be obtained by the limiting approach as described in literature [16]. When q is perturbed by Δq, temperature T undergoes a variation of ΔT. Plugging $q + \Delta q$ and $T + \Delta T$ into Eq. (1), subtracting the Eq. (1) and neglecting the second-order terms, the sensitivity equation (i.e. variation equation) can be obtained as follows:

$$\rho c\frac{\partial \Delta T}{\partial t} = \frac{\partial^2(k\Delta T)}{\partial x^2} + \frac{\partial^2(k\Delta T)}{\partial y^2} + \frac{\partial^2(k\Delta T)}{\partial z^2} \quad \text{for} \quad 0 < x < L,\ 0 < y < M,\ 0 < z < N,\ t > 0 \tag{4a}$$

$$-\frac{\partial(k\Delta T)}{\partial x} = \Delta q(y,z,t) \quad \text{for} \quad x = 0,\ t > 0 \tag{4b}$$