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Transient heat diffusion in multilayered materials with thermal contact resistance

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ABSTRACT

A further extension to the method of recursive images is presented to obtain solutions of the transient diffusion equation in multilayered materials, based on the recursive superposition of Green functions for a semi-infinite material. This extension enables one to find the solution also when thermal contact resistance exists between the layers. Through a sequential sum of image Green functions, a temperature solution is initially built for a structure of one layer over a substrate. These functions are chosen in order to satisfy in sequence the boundary conditions, first at the front interface then at the back interface then again at the front interface and so on until the magnitude of the added functions becomes negligible. This present scheme is now valid for boundary conditions of the first, second and third kind. Four different heat diffusion problems are solved, illustrating how the method works. The first three are diffusion problems of a layer over a substrate while the last one is a three layer over a substrate structure with thermal resistance between layer 2 and 3.

1. Introduction

Recently a new method of recursive images was proposed to solve transient heat diffusion in multilayer materials [1,2]. This method requires a prior knowledge of the analytical semi-infinite solution given the front face boundary condition (BC). The final solution is then found through a sequential sum of Green functions satisfying the BCs, successively at the back and at the front interfaces. In its first formulation only BCs of the first or of second kind were possible to solve.

The method of recursive images was then extended to solve cases, where heat convection took place either at the front or at the back of the multilayer structure. The method could hence solve problems with boundary conditions of third kind and therefore, its applicability expanded into the realm of practical applications. In this paper a further extension to the method is proposed, to enable the method of recursive images to deal with situations where there is thermal contact resistance between layers in a multilayered structure.

Thermal contact resistance plays a key role in heat conduction mechanisms when heat flow is hampered by solid/solid interfaces and is of great technical importance in many areas, especially in electronic devices, in spacecraft and cryogenic applications. The obstruction caused by contact resistances can be extensive, particularly in cases where the heat flow is very high thus creating conditions for the appearance of hot spots. In those cases, special consideration should be given to the surface finish of the contacting faces, the materials which interface, the pressure with which the faces are forced together and the material in the gaps at the interfaces, to reduce any impediments to the heat flow.

There is a wealth of information on modeling and determination of the coefficient of thermal contact resistance [3–11] as well as on the analytical modeling and solution of the transient diffusion in multilayer materials [12–16] where contact resistance is present. Here should be mentioned the work of Powles [17] where exact solutions were given for semi-permeable barriers. The BC suggested for those barriers is in fact that of a thermal contact resistance between two layers of the same material as will be shown below.

This paper has two main sections. In the first, the theory will be introduced describing the Green function applicable to the case of thermal resistance boundary condition, then a short description of the method of recursive images will be given and finally its actual implementation will be exposed. In the second section, four different heat diffusion problems are solved, illustrating how the method works. The first three are diffusion problems with a layer over a substrate configuration while the last one is a three layer structure over a substrate with thermal resistance between layers 2 and 3.

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Nomenclature

x	thickness coordinate	ρ	density (kg/m^3)
d	thickness of a layer (m)	c	specific heat (J/kg)
t	time	h	thermal contact conductance (W/m^2 K)
$T(x,t)$	temperature solution (K)	T_r	one-sided convolution function of an decreasing exponential with a semi-infinite temperature solution
$T_{si}(x,t)$	semi-infinite temperature solution (K)	α_b	parameter of the Green function for a thermal contact resistance which depends only on the thermal properties of the interfacing materials
t_b, t_f	"transmission" strengths		
r_b, r_f	"reflection" strengths		
κ	thermal conductivity (W/m-K)		
D	thermal diffusivity (m^2/s)		

2. Theory

The equation describing diffusion and in particular that of heat diffusion, is a partial differential equation [18,19] given for a multilayer material by,

$$D_i \frac{\partial^2 T_i}{\partial x^2} = \frac{\partial T_i}{\partial t} \tag{1}$$

where D_i is the thermal diffusivity in a layer while T, x and t are the temperature, the thickness direction and time, respectively. The boundary condition for an interface with thermal contact resistance is,

$$\kappa_i \frac{\partial T_i}{\partial x} = \kappa_{i+1} \frac{\partial T_{i+1}}{\partial x} = h_i (T_{i+1} - T_i) \tag{2}$$

where κ is the thermal conductivity and h (W/m^2 K) is the coefficient of thermal contact conductance which is the reciprocal of the thermal contact resistance at an interface i.e. $r_c = 1/h$. Eq. (2) indicate that there is a continuity of heat flow at the interface but there is, in general, no continuity of temperature on both sides of that interface. When h is infinite, however, the temperature on each side of the interface will be equal in order for the rightmost side of Eq. (2) to remain finite. In this latter case the boundary condition will reduce to that of two different materials in intimate contact.

In the work of Powles, of semi-permeable barrier membranes, the same Eq. (2) were used (see Eqs. 1.3 and 1.4 in Powles [17]) except that the "thermal conductivities" were the same on each side of the permeable membrane.

In order to apply the recursive method of images one needs to establish a set of linear operations to be performed whenever there is a thermal wave "incident" at a boundary obeying definite conditions. This is most facilitated through the use of Green functions [1,2,18].

2.1. Green function for an interface between two different media with thermal contact resistance

The Green function for a plane heat source $T_{si}(x_1,t)$, whose origin is at $x_1 = 0$ of medium 1 (see Fig. 1), which interfaces with a semi-infinite back medium at $x_1 = d_1$, has been described by Carslaw [18] (see page 364 and following) for a number of boundary conditions. We first define the following T_{tr} generic function,

$$T_{tr}(T_{si}(x,t);\, h_o) = 2\int_0^{\infty} [h_o e^{-h_o \xi}] T_{si}(x+\xi, t)\, d\xi \tag{3}$$

which represents a one-sided convolution of a semi-infinite temperature solution $T_{si}(x,t)$ with a decreasing exponential which depends on parameter h_o. This function was first established by Bryan [20] (see Eq. (5)) while obtaining the Green function for a convective boundary (see also Carslaw [18] p. 358–359).

The Green temperature function in medium-1 $T_1(x_1,t)$ for the case of a thermal contact resistance boundary condition was first established by Mersman [21] and later reported by Carslaw [18]. It consists, after some manipulation to allow for a change in the length variable, of a sum of the plane heat source $T_{si}(x_1,t)$ with its "reflected image" $T_{si}(2d_1 - x_1,t)$, whose origin is located a distance d_1 to the right from the interface (see Fig. 1), minus a function based on Eq. (3) i.e.,

$$T_1(x_1,t) = T_{si}(x_1,t) + T_{si}(2d_1 - x_1, t) - (1-\alpha_b) T_{tr}\left(T_{si}(2d_1 - x_1, t);\, \frac{h_b/\kappa_1}{1-\alpha_b}\right) \tag{4}$$

where α_b is given by,

$$\alpha_b = \frac{\frac{\kappa_1}{\sqrt{D_1}}}{\frac{\kappa_1}{\sqrt{D_1}} + \frac{\kappa_b}{\sqrt{D_b}}} \tag{5}$$

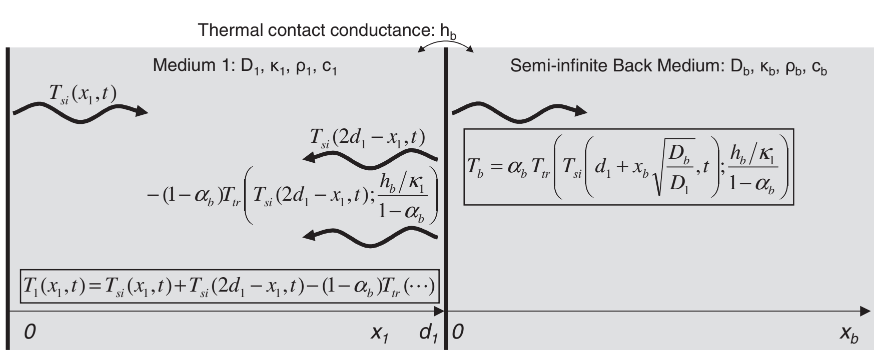

Fig. 1. The Green functions for temperature in two different thermal media assuming a plane source located at $x_1 = 0$ of medium 1. The interface between the media is located in $x_1 = d_1$ ($x_b = 0$). The image of a temperature source at $x = 0$ interfacing at $x = d$ with a back medium with thermal resistance, consists of an image source at $x = 2d_1$ minus a line of instantaneous sinks from $x = 2d_1$ to infinity as described by Eqs. (3) and (4).