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Thermal conduction in an orthotropic sphere with circumferentially varying convection heat transfer

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ABSTRACT

Heat transfer due to fluid flow past a sphere is encountered commonly in engineering applications. In this case, the local convective heat transfer coefficient on the surface of the sphere is known to change with azimuthal and polar angles due to various flow phenomena. While surface-averaged convective heat transfer coefficient is commonly used for engineering analysis, however, not much work exists for modeling the temperature field inside the sphere while accounting for the spatially varying convective heat transfer, especially in the context of a sphere with orthotropic thermal conduction properties. This paper presents an analytical approach for a steady state solution of this problem by deriving a set of algebraic equations for coefficients of a series solution of the temperature distribution. The problem is solved using two different approaches, which are shown to lead to equivalent results. Temperature distribution based on the analytical approach is found to be in excellent agreement with finite-element simulation results. The effect of various parameters, such as thermal conduction orthotropic ratio, heat generation rate, power density, flow rate, etc. on temperature distribution in the sphere is presented. Results discussed in this paper contribute towards the fundamental understanding of an important heat transfer problem, and in the design of thermal management techniques for engineering applications involving convective cooling of spherical systems.

1. Introduction

Fluid flow past a solid body results in energy exchange between the two through convective heat transfer [1–6]. This process is commonly characterized by the convective heat transfer coefficient, h, which, in general, varies over the fluid–solid interface. While a space-averaged heat transfer coefficient is often adopted as an engineering approximation [4], the effect of the spatial variation of convection on temperature fields is clearly important for a complete understanding of this phenomenon [5]. For the specific case of flow past a sphere, the convective heat transfer coefficient is known to vary with both azimuthal and polar angles, φ and θ, respectively. The dependence of h around the periphery of the sphere has been studied for a variety of flow conditions [1,7]. The heat transfer coefficient h is the largest at the stagnation point ($\varphi = 0°$), following which, h first decreases due to laminar boundary layer development. For laminar flow, a minima is reached at around $\varphi = 109°$ where separation occurs [1]. Similarly, h also varies with θ. The temperature and velocity fields in the flow around the sphere have been measured and numerically computed [8]. The overall heat transfer coefficient has been measured for small spheres in a fluid flow and a relationship between heat transfer, flow velocity and fluid properties has been derived using experimental data [9]. An analytical solution for transient heat transfer from a sphere at low Reynolds number under steady velocity conditions has been developed [10]. Analytical solution for unsteady heat transfer at small Peclet numbers has also been developed when the surface temperature of the sphere undergoes a step change [11].

Most of this past work addresses temperature and velocity fields in the fluid, whereas the temperature field within the sphere, and its dependence on the φ and μ dependent convective heat transfer coefficient has not been adequately addressed in the literature. Such analysis has been carried out in the past for other geometries, including extended surfaces [12–15] and orthotropic cylinders [5], using a variety of analytical techniques to account for the space-dependent convective heat transfer. A Fourier series method has been used to determine the two-dimensional temperature distribution in a rectangular fin with heat transfer coefficient varying along its length [12]. Temperature in fins with varying geometries and heat transfer coefficient has been computed using Frobenius series expansion method [13]. The performance of

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Nomenclature

h	convective heat transfer coefficient, (W/m^2-K)	R	sphere radius, (m)
k	thermal conductivity, (W/m-K)	T	temperature rise above ambient, (K)
Q	volumetric heat generation rate, (W/m^3)	θ	polar angle
r	radial coordinate, (m)	φ	azimuthal angle
P_n^m	associated Legendre functions of degree n and order m		

annular fins in the presence of variable heat transfer coefficient has been studied [14]. A Galerkin based integral approach has also been adopted to account for variable heat transfer coefficient in fins [15]. An analytical solution for temperature distribution in an orthotropic cylinder with spatially-varying heat transfer coefficient on its surface has been presented [5] by assuming a Fourier series form of the temperature distribution with coefficients that are determined using the spatial variation of the heat transfer coefficient. In addition to such analytical approaches, numerical methods have also been used to analyze cases with space-varying heat transfer coefficient. For example, a finite difference based model has been adopted to study the transient heat transfer of a solid sphere in cross flow [16].

This paper presents an analytical derivation to compute the temperature distribution in a sphere with spatially varying convective heat transfer coefficient on its surface. Thermal conduction within the sphere is assumed to be orthotropic in general, with different thermal conductivity values in r, φ and θ directions. Volumetric heat generation occurs within the sphere, which is cooled on the outside surface with a convective heat transfer coefficient that depends on both azimuthal and polar angles. A Fourier series form of the temperature distribution is assumed. It is shown that the series coefficients can be determined by solving a set of linear algebraic equations that account for the general spatial variation of h on the sphere surface. The temperature distribution computed by this analytical solution is found to be in good agreement with results from finite-element simulations. The dependence of the temperature profile on a number of parameters such as the heat transfer coefficient, thermal conduction orthotropy, etc. is discussed.

The theoretical derivation of temperature field in a sphere with orthotropic thermal properties is important because while most commercial finite-element simulation tools enable analysis of orthotropic thermal conduction in rectangular and cylindrical coordinate systems, the treatment of orthotropic thermal conduction in spherical coordinates is not available. By deriving the temperature distribution for this very general case, the treatment presented here may help expand the capability of thermal analysis in spherical coordinate systems.

2. Analytical model

This section presents the derivation of the steady state temperature distribution in an orthotropic sphere with internal volumetric heat generation and spatially dependent h. Based on the general derivation presented next in Section 2.1, a special case for a partially orthotropic sphere where $k_\varphi = k_\mu$ is presented in Section 2.2. Section 2.3 discusses an alternate analytical approach for solving the general problem. Finally a brief discussion is presented, showing that for isotropic conditions, i.e. all thermal conductivities being the same, the solutions presented for the orthotropic and partially orthotropic cases reduce to that of the isotropic solution as one would expect.

2.1. Orthotropic sphere

Fig. 1 shows a schematic of the general heat transfer problem being addressed in this sub-section. The steady-state governing energy equation in a three dimensional orthotropic sphere is given by [17]

$$k_r\left[\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right]+\frac{k_\phi}{r^2(1-\mu^2)}\frac{\partial^2 T}{\partial \varphi^2}+\frac{k_\mu}{r^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial T}{\partial \mu}\right]+Q=0 \tag{1}$$

where $T(r, \varphi, \mu)$ is the temperature rise above ambient, $\mu = \cos(\theta)$, k_r, k_φ and k_μ are thermal conductivities in the r, φ and μ directions, and Q is the volumetric heat generation rate.

The temperature distribution is subject to the following boundary conditions:

$$\frac{\partial T}{\partial r}=0 \quad \text{at } r=0 \tag{2}$$

$$k_r\frac{\partial T}{\partial r}+h(\varphi,\mu)\cdot T=0 \quad \text{at } r=R \tag{3}$$

$$T(r,\varphi,\mu)=T(r,\varphi+2\pi,\mu) \tag{4}$$

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi}=\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi+2\pi} \tag{5}$$

Eq. (2) represents the requirement for the temperature field to be finite at $r = 0$. The circumferential variation of h at $r = R$ is accounted for by Eq. (3). Eqs. (4) and (5) represent temperature periodicity and heat flux continuity in the φ-direction. In addition to satisfying Eqs. (1)–(5), the temperature field must also remain bounded in the μ direction [17].

If h were a constant number, then the solution for Eqs. (1)–(5) can be obtained by the separation of variables method in a

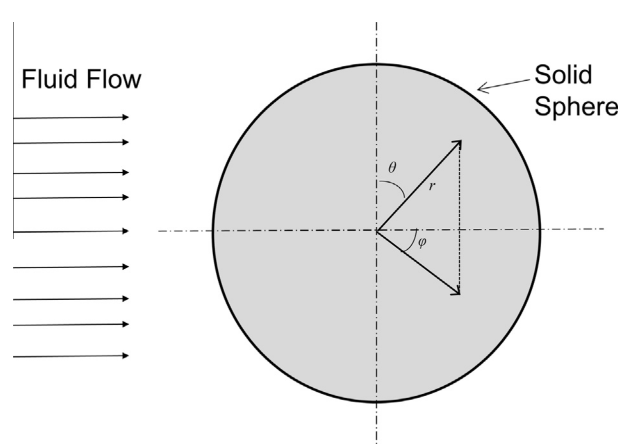

Fig. 1. Schematic of the problem.