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On three-dimensional flow of nanofluids past a convectively heated deformable surface: A numerical study

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ABSTRACT

This paper is concerned with the three-dimensional rotating flow of nanofluid induced by a convectively heated deformable surface. The base fluid is treated as water while three different types of nanoparticles namely copper oxide-CuO, copper-Cu and silver-Ag are analyzed. Two different models for effective thermal conductivity of nanofluids are employed. A self-similar form of the governing differential system is formulated via adequate transformations. Shooting approach combined with fifth order Runge–Kutta method is used to determine the velocity and temperature distributions above the sheet. Our computations reveal that skin friction coefficient has direct relationship with the volume fraction of nanoparticles. Further the surface heat transfer rate is an increasing function of the solid volume fraction of nanoparticles. Sketch for three-dimensional streamlines is also obtained and discussed for a particular case.

1. Introduction

Rotating flows are involved in several fascinating applications including geothermal extraction, food processing, polymer processing through chemical mixing chambers, thermal oil recovery process, viscometry and many others. The pioneering study on the rotating flow induced by a stretching surface was presented by Wang [1]. He indicated that such flow problem is important in understanding the geological stretching of tectonic plate beneath a rotating ocean. He constructed the self-similar solutions by regular perturbation approach. Rajeswari and Nath [2] discussed the unsteady rotating flow over a stretching sheet by finite difference approach followed by the quasilinearization technique. Later, Nazar et al. [3] addressed the unsteady flow induced by a stretching surface in a rotating fluid. Analytic solutions for rotating flow of non-Newtonian second grade fluid were provided by Hayat et al. [4]. Abbas et al. [5] extended the work of Nazar et al. [3] with the inclusion of magnetohydrodynamics and heat transfer effects. They presented numerical solutions by Keller-box method. Zaimi et al. [6] also used Keller-box method to investigate the rotating flow of elastico-viscous fluid developed by a stretching surface. Turkyilmazoglu [7] presented an interesting study dealing with the three-dimensional motion of fluid at sufficiently large distance from a uniformly stretching disk. Mustafa [8] studied the Cattaneo–Christov heat flux model for rotating flow of Maxwell fluid bounded by a stretching surface. He obtained both analytical and numerical solutions for the formulated non-linear differential system.

Nanofluids are the new generation heat transfer fluids engineered by uniform and stable suspension of nanometer-sized particles in the base fluids. These fluids possess much higher thermal conductivity in comparison to the conventional heat transfer fluids even at very low particle concentrations [9]. The enhanced thermal conductivity of nanofluids is beneficial for a number of technical applications including solar energy absorption, drug delivery, nuclear engineering, space cooling, cooling of microelectronics, power generation, transportation and many others. Two successful models for convective transport in nanofluids are available in the literature namely the Tiwari and Das model [10] and Buongiorno's model [11]. Tiwari and Das model [10] focuses on the heat transfer enhancement via solid volume fraction of different nanoparticles and base fluids. On the other hand Buongiorno's model addresses the important aspects of Brownian motion and thermophoresis. Nield and Kuznetsov [12] employed Buongiorno's model to explore the Cheng–Minkowycz problem for natural convective flow of nanofluid through a porous space. In another paper, Nield and Kuznetsov [13] discovered the natural convective flow of nanofluid past a vertical isothermal plate. The fundamental study on the boundary layer flow of nanofluid developed by stretching surface was addressed by Khan and Pop [14]. They employed Keller-box

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method to simulate the governing non-linear differential system. Mustafa et al. [15] analytically explored the classical problem of two-dimensional stagnation-point flow along a stretching surface. Uddin et al. [16] investigated the free convective flow of nanofluid past a vertical plate considering more realistic Newtonian heating boundary conditions. Sheikholeslami [17] used Tiwari and Das model to discover the influence of magnetic field on the two-dimensional flow of Cu–water nanofluid. Turkyilmazoglu [18] analyzed the classical Von Karman flow for five different types of water based nanofluids. Recently a variety of flow problems involving nanofluids have appeared in the literature [19–30].

Present work focuses on the rotating flow of nanofluids induced by a convectively heated stretching surface through Tiwari and Das model [10]. Water based nanofluids containing three different nanoparticles namely copper oxide-CuO, copper-Cu and silver-Ag are considered. In addition, two different models for thermal conductivity of nanofluids are also employed. The formulated differential system is treated through a numerical approach. Velocity and temperature distributions have been carefully analyzed by plotting graphs. Numerical results for skin friction coefficients and local Nusselt number are obtained and discussed.

2. Mathematical model

Consider a three-dimensional rotating flow of nanofluid over a surface coincident with the plane $z = 0$. The surface is subjected to uniform stretching in the x-direction with the velocity $u_w = ax$ where $a > 0$ is constant. The flow is composed of three different nanoparticles namely copper oxide-CuO, copper-Cu and silver-Ag. The temperature at the sheet is passively controlled via convection from hot fluid (below the surface) at temperature T_f (see Fig. 1). Let T_∞ be the temperature outside the thermal boundary layer. Using the standard boundary layer approximations, the equations governing the conservation of mass, momentum and energy are expressed as below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v\right) = \mu_{nf}\left(\frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\Omega u\right) = \mu_{nf}\left(\frac{\partial^2 v}{\partial z^2}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial z^2}\right), \tag{4}$$

with the following boundary conditions

$$u = u_w = ax, \quad v = 0, \quad w = 0, \quad -k_{nf}\frac{\partial T}{\partial z} = h_f(T_f - T) \text{ at } z = 0,$$
$$u \to 0, \quad v \to 0, \quad T \to T_\infty \text{ as } z \to \infty, \tag{5}$$

where u, v and w are the velocity components along the x-, y- and z-directions respectively, T is the fluid temperature, ρ_{nf} is the density of the nanofluid, μ_{nf} is dynamic viscosity of the nanofluid and α_{nf} is thermal diffusivity of the nanofluid. These are defined as below:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{6}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \tag{7}$$

in which ϕ denotes the nanoparticle volume fraction, ρ_f and ρ_s are the densities of the base fluid and nanoparticle material respectively, k_f and k_s are the thermal conductivities of the base fluid and nanoparticle material respectively and $(\rho c_p)_{nf}$ is the effective heat capacity of the nanofluid. The thermophysical properties of water and nanoparticle materials are given in Table 1. It should be noted here that thermal conductivity of nanofluid k_{nf} given above is termed as Maxwell–Garnett model [31]. This approximation is valid for spherical nanoparticles. For a two-component entity of spherical-particle suspension, Patel et al. [32] proposed the following model.

$$\frac{k_{nf}}{k_f} = \left(1 + (1 + cPe)\frac{k_s}{k_f}\frac{A_s}{A_f}\right), \tag{8}$$

where the values of the constants c, Pe, A_s and A_f are taken from [33]. We consider both the models for our computational analysis.

Introducing the following similarity transformations [6]

$$\eta = \sqrt{\frac{a}{\nu_f}}z, \quad u = axf'(\eta), \quad v = axg(\eta),$$
$$w = -\sqrt{a\nu_f}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{9}$$

the continuity equation (1) is automatically satisfied while Eqs. (2)–(5) reduce to

$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)}f''' - f'^2 + ff'' + 2\lambda g = 0, \tag{10}$$

$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)}g'' + fg' - f'g - 2\lambda f' = 0, \tag{11}$$

$$\frac{k_{nf}/k_f}{\left(1-\phi+\phi(\rho c_p)_s/(\rho c_p)_f\right)}\frac{1}{\Pr}\theta'' + f\theta' = 0, \tag{12}$$

Table 1
Thermophysical properties of water and some nanoparticles.

Properties	Fluid (water)	CuO	Cu	Ag
k (W/m K)	0.613	76.5	400	429
ρ (kg/m^3)	997.1	6320	8940	10,500
c_p (J/kg K)	4179	531.5	385	233

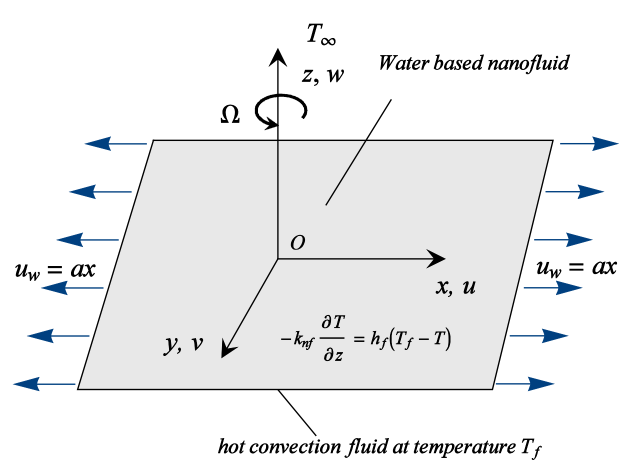

Fig. 1. Physical configuration and coordinate system.