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Influence of magnetic field on the periodically oscillating fluid inside a porous medium attached to a thick solid plate

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ABSTRACT

This paper presents a rigorous mathematical analysis of the influence of a magnetic field on a periodically oscillating fluid inside a porous medium. We consider a porous medium coupled with a thick solid plate with a magnetic field perpendicular to the direction of fluid oscillations. The hydrodynamic and thermal interactions of the oscillating fluid with the porous medium and the thick solid plate are modeled analytically as a thermoacoustic system under the influence of a transverse magnetic field. The velocity and temperature expressions of the oscillating fluid are derived using the perturbation technique after simplifying the governing Darcy momentum and energy equations. From the flow and thermal fields' results, Nusselt number, heat flux, and work flux are calculated and presented graphically. Consequently, the entropy generation rate for the overall system is investigated to assess the irreversibility associated with the proposed system enabling one to improve the efficiency of the system. Finally, the efficiency of the proposed thermoacoustic system is determined using the expressions of heat and work fluxes. It is observed that the thermoacoustic irreversibility can be minimized by increasing the applied magnetic force resulting in increased efficiency of the proposed system.

1. Introduction

The thermal interaction between compressible oscillating fluids and solid surfaces produce thermoacoustic effects [1]. Such effects can cause simultaneous velocity, pressure, and temperature fluctuations [1]. A thermodynamic cycle is evolved in the vicinity of the solid surfaces due to the thermoacoustic effect and the consequence of this cycle is the conversion of energy from thermal to acoustic or from acoustic to thermal.

A thermoacoustic system possesses certain advantages. These advantages include no moving components, low maintenance cost, environmental friendly working medium (e.g., inert gases), and the use of low potential energy input sources [2]. Therefore, thermoacoustic related research topics have caught the attention of numerous researchers over the past few decades [2–7]. The temperature gradient, required to run a thermoacoustic system, can be developed using any low potential sources of energy; for example, solar energy [8], waste heat from automotive engine [9], and industrial waste heat [10]. A wide variety of combustible fuels can also be used to run a thermoacoustic system. For example, natural gas, bio-fuel, methane, alcohol, gasoline, and fuel oil.

Thermoacoustic effects can be effectively utilized to produce different systems. For example, thermoacoustic heat engine [11,12], refrigerators [13], gas mixture separators [14], electrical energy generators [15], heat exchangers [16], imaging systems for tumor detection [17], and breast cancer detection systems [18].

A thermoacoustic device typically consists of three major elements: a stack, a cold heat exchanger, and a hot heat exchanger. The heat exchangers are attached to both ends of the stack to transfer heat to and from the stack. These three elements are usually placed inside a resonant tube filled with air or inert gases. A temperature gradient can be created across the stack by using appropriate thermal loads on the heat exchangers. Fluid inside the resonant tube starts oscillating if this temperature gradient exceeds a critical value [2]. Such fluid oscillation will create thermoacoustic wave if it exceeds the frictional and other losses inside the resonant chamber. This wave can be converted into other forms of energy (e.g., electricity) using proper transducers (e.g., piezoelectric transducer) [15]. On the other hand, if acoustic energy is used as an input to a thermoacoustic system, a temperature difference between the two heat exchangers can be obtained if the temperature gradient across the stack is lower than a critical value [19]

Although the thermoacoustic system has many inherent advantages, poor efficiency is still the major drawback of the

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Nomenclature

B_y	vertical component of magnetic induction, Wb m^{-1}
C_p	specific heat of the fluid, J kg^{-1} K^{-1}
C_{sm}	specific heat of the solid matrix, J kg^{-1} K^{-1}
Da	ratio of permeability of the porous medium and viscous penetration depth, $= K/\delta_v^2$
DR	drive ratio, $= p_0/p_m$
E	electrical field intensity, Volt m^{-1}
$\mathbf{F_{em}}$	electromagnetic volume force, N m^{-3}
f	frequency of oscillation, Hz
Ha_δ	Hartmann number, $= B_y \delta_v \sqrt{\sigma_k/\mu}$
i	complex number, $= \sqrt{-1}$
$\mathbf{J}$	current density, amp m^{-2}
K	permeability of the porous medium, m^2
k_s	solid wall thermal conductivity, W m^{-1} K^{-1}
k_{sm}	thermal conductivity of solid matrix, W m^{-1} K^{-1}
k_f	thermal conductivity of the fluid, W m^{-1} K^{-1}
k	effective thermal conductivity, W m^{-1} K^{-1}
Nu	complex Nusselt number
p	pressure, N m^{-2}
p_m	mean pressure, N m^{-2}
∇p	pressure gradient, N m^{-3}
p_0	fluctuating pressure amplitude, m
Pr	Prandtl number of the fluid,$= \delta_v^2/\delta_k^2$
$\dot{Q}_2$	second order local heat flux, W m^{-3}
$\dot{s}'''_{gen}$	volumetric entropy generation rate, W m^{-3} K^{-1}
$\dot{S}_{gen}$	total entropy generation, W K^{-1}
S	dimensionless entropy generation
T	temperature of the fluid, K
T_{s1}	temperature inside the solid wall, K
∇T	temperature gradient, K m^{-1}
t	time, s
u	Axial velocity component, m·s^{-1}
V	Velocity vector, m·s^{-1}
$\dot{w}_2$	second order work flux, W·m^{-3}
x	axial direction, m
Δx	length of the porous medium and solid wall, m
y	transverse direction of the porous medium, m
$\bar{y}$	transverse direction of the solid wall, m

Greek Symbols

σ	porous medium heat capacity ratio (see. Eq. (8a))
σ_k	electrical conductivity of the fluid
ϕ	porosity (=void volume/total volume)
Γ	ratio of ∇T_m and ∇T_{cr}
Γ^*	ratio of ∇T_m and ∇T_{cr} in the absence of magnetic field, $= \nabla T_m K \rho_m C_p / T_m \beta \mu \lambda f$
ρ	density of the fluid, kg m^{-3}
ρ_m	mean density of fluid, kg m^{-3}
ρ_{ms}	mean density of the solid, kg m^{-3}
ρ_{sm}	density of the Solid matrix, kg m^{-3}
Ψ	viscous dissipation function
α_f	thermal diffusivity of the fluid, m^2 s^{-1}
α_s	thermal diffusivity of the solid wall, m^2 s^{-1}
β	thermal expansion coefficient, K^{-1}
δ_v	viscous penetration depth, $= \sqrt{2\nu/\omega}$
δ_k	thermal penetration depth, $= \sqrt{2\alpha_f/\omega}$
δ_s	thermal penetration depth, $= \sqrt{2\alpha_s/\omega}$
ρ	density of the fluid, kg m^{-3}
τ	time period, $= 2\pi/\omega$
η	efficiency
λ	acoustic wavelength, m
ε	solid wall to porous medium heat capacity ratio $= \sqrt{\rho_m C_p k_f/\rho_s C_s k_s}$
Π	width of the porous medium and solid wall, m
$\Re[\,]$	real part
∇	gradient
$\sim$	complex conjugate
$-$	time average value

Subscripts and superscripts

1	first order variable
∞	free stream value
m	mean value
sm	properties correspond to solid matrix material of the porous medium
w	value at the interface of the solid wall and porous medium
cr	critical value
a	adiabatic oscillation
FF	fluid friction
HT	heat transfer
s1	value inside the solid wall
Mag	magnetic force

thermoacoustic system. The thermoacoustic efficiency varies between 20% and 30% of the Carnot efficiency [20]. In comparison to the conventional heat pump or heat engine, thermoacoustic systems also have high power density.

Different approaches have been found in the literature to increase the power density [21,22] and efficiency [23] of the thermoacoustic systems, such as, Swift [2] found that if the stack material is poorly thermal conductive, it will achieve proper phasing between the oscillating fluid and the solid wall and eventually increase the efficiency. Till now the choice of stack material is very limited. For example, the wire mesh stack, the plastic roll stack, a metal or ceramic honeycomb stack having square and hexagonal channel sections, and the pin stack [24]. Each type of stack configuration has its own advantages and disadvantages.

Adeff et al. [25] reported after performing experiments that using reticulated vitreous carbon (RVC) as a stack material instead of a traditional plastic roll stack increased the performance of the thermoacoustic prime mover and refrigerator. The RVC is a poorly conductive, highly porous, has higher specific heat, relatively cheaper, lightweight, and easy to machine material. However, the major disadvantage of RVC is its higher brittleness. Adeff et al. [25] did not develop theoretical model for RVC as a stack material. However, they verified their experimental results with Swift's [2] general theories of thermoacoustic and found good agreement.

Roh and Raspet [26] developed thermoacoustic theories for a random porous medium. Their theories included development of thermal and viscous functions, wave, and temperature distribution equation. Jensen and Raspet [27] developed an analytical model for fibrous porous materials. Mahmud and Fraser [28] extended thermoacoustic theories that approximated stack as a porous medium embedded inside two thin solid walls. They used transient Darcy–Brinkman momentum equation to model the fluctuating velocity inside the porous medium. Mahmud and Fraser [28] developed analytical model for the wave equation, fluctuating velocity, temperature, complex Nusselt number, and energy flux. Mahmud and Pop [29] extended Mahmud and Fraser's [28] work by identifying modes of operations by observing the energy field variation with the dimensionless Darcy number. Mahmud and Pop [29]